Modeling outbursts of viscous accretion discs

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Abstract. A brief review is given on the topic of viscously-evolving accretion discs around compact objects that covers the development of analytical studies and our numerical model `freddi` allowing comparison of theory with observations.

Keywords: accretion, accretion discs

1 Introduction

The most suitable sources to compare observations with predictions of the theory of viscously evolving accretion discs are X-ray novae with black holes (BHs), when emission of the disk dominates during flares, if a source is in a ‘soft’ state. An outburst of an X-ray novae typically lasts from a couple of tens of days to few months and can have quite different shapes. Among all the variety of outburst profiles, there are so-called FRED flares (’fast-rise exponential-decay’). They are of particular interest for theoretical study.

Although there is still a lot to be learned about spectra formation and variability, and mechanisms causing peculiarities of light curves, the general properties of FRED outbursts are reliably explained in the theory of a viscous disk.

2 Theoretical advance

The equation of the viscous disc evolution that governs its long-term dynamics (see, e.g., Lyubarskij & Shakura 1987)

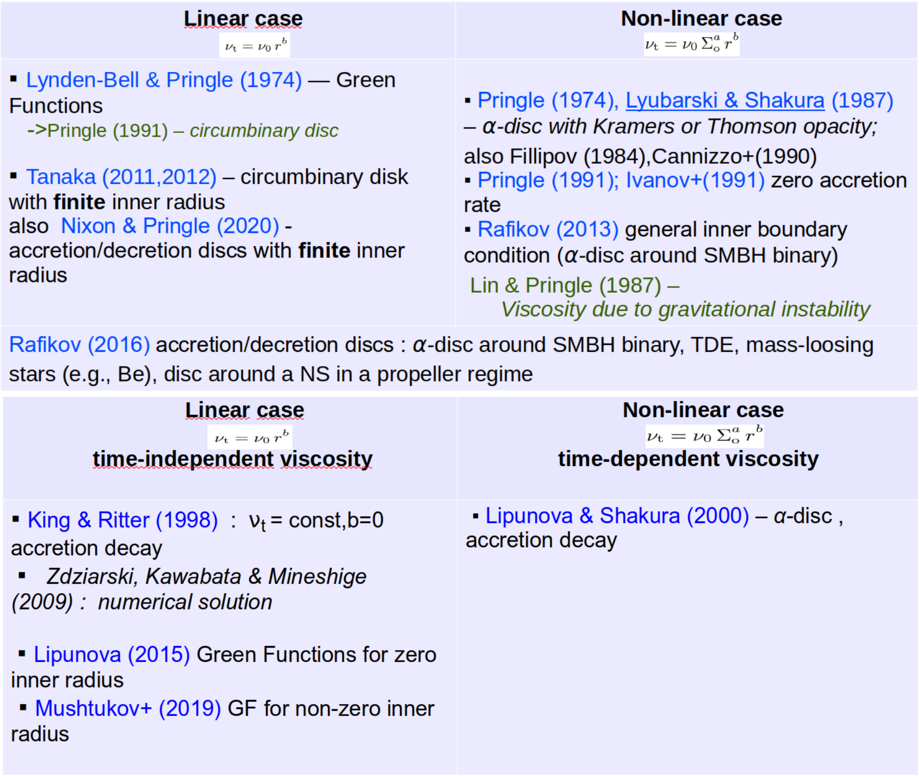
$$\frac{\partial \Sigma}{\partial t} = \frac{1}{4\pi} \frac{(GM_\star)^2}{h^3} \frac{\partial^2 F}{\partial h^2}, \tag{1}$$

is derived from equations of conservation of mass and angular momentum (see also Lynden-Bell & Pringle 1974, LP74). Since the suitable geometry for the disc is a cylindrical one, all values are integrated over the disc height: the surface density Σ and the viscous torque $F = 2\pi\, r^2 W_{r\varphi}$, where $W_{r\varphi} = \frac{3}{2}\,\omega_{\rm K} \nu_{\rm t} \Sigma$ is the integrated component of the viscous stress tensor. Note that the radial coordinate is substituted by the specific angular momentum $h = \sqrt{GM_\star r}$. This change of variable allows one to deal with a simplified form of the equation as well as to pose boundary conditions in the most suitable way.

Turbulent motions, believed to be generated by the MHD instabilities, sustain the viscosity of astrophysical accretion discs. The kinematic coefficient of viscosity $\nu_{\rm t}$ is a product of the characteristic length and velocity of turbulent motions and can be related to the α-parameter of the standard model (Shakura & Sunyaev 1973; Shakura et al. 2018).

A list of analytic solutions of Eq. (1) is given in Fig. 1. There are two groups of solutions (see also Fig. 2): when the kinematic coefficient of viscosity $\nu_{\rm t}$ depends only on the radius (the left column) and when it also depends on the hydrodynamic parameters (the right column; this case is of a special importance since α-discs falls into this category). When Eq. (1) is a linear differential equation, Green functions (GF) is an effective method to solve it. LP74 in their foundational work found GF for a disc with a zero inner radius and infinite outer radius. Later, Tanaka (2011) and Nixon & Pringle (2020) found GF when the inner boundary is at the finite radius. GF for a disc with finite outer radius are given by Lipunova (2015) and Mushtukov et al. (2019). The latter have been used to build a power density spectra of X-ray variability generated by mass accretion fluctuations over the disc around a magnetized neutron star (Mushtukov et al. 2019).

King & Ritter (1998) (KR98) proposed a model with constant $\nu_{\rm t}$ to explain the exponential evolution of X-ray nova FREDs; Lipunova & Shakura (2000) (LS00) showed that in an α-disc the accretion rate evolves not exponentially but as a power-law, although it is observed in X-rays to be very close to exponential. Figure 2 compares the solutions of the linear and non-linear Eq. (1) with similar absolute values of ν_t. It is evident that during few viscous characteristic (exponential) times these solutions are very close.



Linear case $\nu_t = \nu_0\, r^b$	**Non-linear case** $\nu_t = \nu_0\, \Sigma_o^a\, r^b$
▪ Lynden-Bell & Pringle (1974) — Green Functions ->Pringle (1991) – *circumbinary disc* ▪ Tanaka (2011,2012) – circumbinary disk with **finite** inner radius also Nixon & Pringle (2020) - accretion/decretion discs with **finite** inner radius	▪ Pringle (1974), Lyubarski & Shakura (1987) – *α-disc with Kramers or Thomson opacity;* also Fillipov (1984),Cannizzo+(1990) ▪ Pringle (1991); Ivanov+(1991) zero accretion rate ▪ Rafikov (2013) general inner boundary condition (α-disc around SMBH binary) Lin & Pringle (1987) – *Viscosity due to gravitational instability*
Rafikov (2016) accretion/decretion discs : α-disc around SMBH binary, TDE, mass-loosing stars (e.g., Be), disc around a NS in a propeller regime	

Linear case $\nu_t = \nu_0\, r^b$ **time-independent viscosity**	**Non-linear case** $\nu_t = \nu_0\, \Sigma_o^a\, r^b$ **time-dependent viscosity**
▪ King & Ritter (1998) : ν_t = const,b=0 accretion decay ▪ *Zdziarski, Kawabata & Mineshige (2009) : numerical solution* ▪ Lipunova (2015) Green Functions for zero inner radius ▪ Mushtukov+ (2019) GF for non-zero inner radius	▪ Lipunova & Shakura (2000) – α-disc , accretion decay

Fig. 1. Analytic solutions for freely expanding (top) and radially confined (bottom) discs.

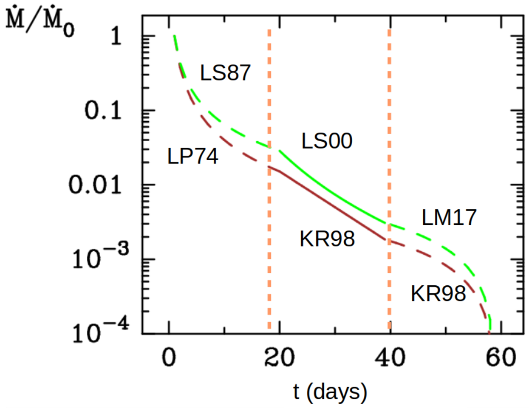


Fig. 2. Analytic solutions for discs with constant ν_t (brown line) and α-discs (green line). In interval A, discs are freely-expanding over radius. In a binary, there is a stage when a disc 'feels' its outer boundary: interval B. Curves in interval C depicts accretion rate evolution when the hot part of a disc shrinks.

3 Observations vs. theory

Analysing observed light curves of X-ray novae, one can in principle infer the value of the α-parameter (Smak 1999; Lipunova & Shakura 2002; Suleimanov et al. 2008; Lipunova & Malanchev 2017; Tetarenko et al. 2018). It turns out that self-irradiation has impact not only on the observed optical flux but also on the course of the evolution. Thus, a combined analysis of observations in X-ray and optical is necessary to build a physically-consistent model of an accretion disc. To determine reliably α-parameter, the binary parameters (masses, inclination) are need to be known quite accurately.

Furthermore, a spectral analysis in X-ray band is desirable, since we need to separate the disc flux from other spectral contributions to derive the central accretion rate variation in order to compare it with a model. If non-thermal components in spectra are bright and evolving, jets are contributing, the analysis becomes very involved.

It must be noted that analytic solutions are not applicable to discs with non-uniform type of viscosity. For example, if a disc is large, the matter in its outer part is not ionised and quite 'cold' (temperature $\lesssim (1-3)\,10^4$ K), and the viscous time is considerably larger there. The boundary, or transition zone, between 'hot' and 'cold' part of the disc moves during an outburst.

To numerically model the disc evolution and to take into account described components of the model, we have developed an open code `freddi` (Malanchev & Lipunova 2016). Its original version dealt with accretion discs around black holes (Lipunova & Malanchev 2017). Recently, accretion discs around neutron stars are incorporated in `freddi` and applied to an outburst of Aql X-1, a LMXB with a NS (Lipunova et al. 2021). The code allows one to include a thermal wind from its surface (Avakyan et al., in press). It calculates the evolution of the disc radial structure and light curves in user-specified bands. Optical flux can be calculated taking into account irradiation of the companion star.

4 Acknowledgments

This work is supported by the Russian Science Foundation under grant 21-12-00141 and performed in Moscow Lomonosov State University. NS acknowledges support of the Interdisciplinary Scientific and Educational School of Moscow University "Fundamental and Applied Space Research". ST acknowledges support by the grant 14.W03.31.0021 of the Ministry of Science and Higher Education of the Russian Federation.

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