Astronomy & Astrophysics manuscript no.
(will be inserted by hand later)

Period distribution of old accreting isolated neutron stars

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Abstract. In this paper we present calculations of period distribution for old accreting isolated neutron stars (INSs). After a few billion years of evolution low velocity INSs come to the stage of accretion. At this stage the INS's period evolution is governed by magnetic braking and the angular momentum accreted. Since the interstellar medium is turbulent the accreted momentum can either accelerate or decelerate the spin of an INS, therefore the evolution of the period has a chaotic character. Our calculations show that in the case of constant magnetic field accreting INSs have relatively long spin periods (some hours and more, depending on the INS's spatial velocity, its magnetic field and the density of the surrounding medium). Such periods are much longer than the values measured by *ROSAT* for 3 radio-silent isolated neutron stars. Due to their long periods INSs should have high spin up/down rates, $\dot{p}$, which should fluctuate on a time scale of ~ 1 yr.

Key words. neutron stars – magnetic fields – stars: magnetic field – X-rays: stars – accretion

1. Introduction

The spin period is one of the most precisely determined parameters of a neutron star (NS). Estimates of masses (for isolated objects), radii, temperatures, magnetic fields etc. are usually model dependent. That is why it is very important to have a good picture of evolution of the only model independent physical parameters of INSs — the spin period and its derivative, as they are usually used to determine other characteristics of NSs. Our aim is to obtain the distribution of periods p for old accreting INSs (AINSs), we also briefly discuss possible values of period derivative $\dot{p}$.

A lot has been done to understand the period evolution of radio pulsars (see Beskin et al. 1993) and NSs in close binaries (Ghosh & Lamb 1979, Lipunov 1992). AINS are especially interesting from the point of view of period and magnetic field evolution since their history is not "polluted" by huge accretion, as happens with their relatives in close binary systems, where NSs can accrete up to $1\,M_{\odot}$ during extensive mass transfer.

In early $90^{\rm s}$ there has been a great enthusiasm about the possibility to observe a huge population of AINSs with *ROSAT* (Treves & Colpi 1991, Blaes & Rajagopal 1991, Blaes & Madau 1993, Madau & Blaes 1994). However, it has become clear that AINSs are very *elusive* (Treves et al. 1998) due to their high spatial velocities (**Neühauser & Trümper 1999**, Popov et al. 2000a) or/and magnetic field properties (Colpi et al. 1998, Livio et al. 1998).

Still, AINSs (and radio-silent INSs in general) are a subject of interest in astrophysics (see Caraveo et al. 1996, Treves et al. 2000 and references therein). A few candidates seem to be observed by *ROSAT* (Motch 2001), although it is also possible that these sources (at least a part of them) can be better explained by young cooling NSs (see Neühauser & Trümper 1999, Yakovlev et al. 1999, Walter 2001, Popov et al. 2000b). Nevertheless, such sources should be very abundant at the low fluxes attainable by *Chandra* and *XMM–Newton* observatories. In (Popov et al. 2000b) the authors obtain that at $\sim 10^{-13}$ erg s^{-1} **cm**$^{-2}$ AINSs become more abundant than young cooling NSs, and the expected number is about 1 source per square degree for fluxes $> 10^{-16}$ erg s^{-1} **cm**$^{-2}$. So, the calculation of properties of AINSs is of great importance now.

Previous estimates of the spin properties of AINSs (Lipunov & Popov 1995a, Konenkov & Popov 1997) have given only typical values of periods, but no realistic distributions of this parameter are calculated. "Spin equilibrium" of NSs with the interstellar medium (ISM) has been

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always assumed, i.e. the authors have considered the situation, when all INS have enough time for spin evolution, and they have not considered NSs with relatively high spatial velocities. In this paper we present full analysis of the problem.

Period estimates are especially important as this parameter can be used to distinguish accreting INSs from young cooling NSs and background objects.

We proceed as follows: In the next section we describe the model used to calculate the period distribution. In Section 3 we present our results. In this paper we do not address the question of the total number of AINS, for this data we refer to our previous calculations (Popov et al. 2000a, Popov et al. 2000b). Here we only show period distributions for AINSs. In the last section we give a brief discussion, derive typical parameters of $\dot{p}$ for AINSs and summarize the paper.

2. Model

In this section we describe our model of spin evolution of AINS in a turbulent ISM. We consider constant ISM density and isotropic Kolmogorov turbulence[1] (a Kolmogorov spectrum is in good correspondence with most observations of interstellar, i.e. intercloud, turbulence, see for example Falgarone & Philips 1990). The orientations of turbulent cells at all scales are assumed to be independent. Turbulent velocities at different scales relate to each other according to the Kolmogorov law:

$$\frac{v_t(r_1)}{v_t(r_2)} = \left(\frac{r_1}{r_2}\right)^{1/3} .$$

Observations show (see Ruzmaikin et al. 1988 and references therein) that the turbulent velocity at the scale of $R_t \simeq 2 \cdot 10^{20}$ cm $\simeq 70$ pc is about $v_t \simeq 10$ km s^{-1}. The above scale is close to the thickness of the gas disk of the Galaxy, and the above velocity — to the speed of sound in the ISM. It corresponds to the largest cell size possible and the fastest movements (otherwise turbulence will efficiently dissipate its energy in shocks).

As an AINS moves through the ISM it can capture matter inside the so called Bondi (or accretion, or gravitational capture) radius, $R_G = 2GM/v^2$. Here G is the gravitational constant, M is the mass of NS, $v = \sqrt{v_{NS}^2 + v_s^2}$, v_{NS} is the spatial velocity of a NS relative to ISM, v_s is the speed of sound (we assume $v_s = 10$ km s^{-1}, $M = 1.4\,M_\odot$ everywhere in the paper; v_s can be dependent on the luminosity of AINS (Blaes et al. 1995), but here we neglect it, but plan to include this dependence in our future calculations).

The accretion rate $\dot{M}$ at the conditions stated above is equal to (Hoyle & Littleton 1939, Bondi & Hoyle 1944):

$$\dot{M} = \pi R_G^2 n m_p v ,$$

here n is the number density of the ISM, m_p is the mass of proton. This accretion rate corresponds to the luminosity:

$$L = GM\dot{M}/R \sim 10^{32} n\,(v/10\,\mathrm{km\,s^{-1}})^{-3}\,\mathrm{erg\,s^{-1}}$$

Based on population synthesis models (Popov et al. 2000b) we can expect, that on average AINS should have luminosities of about 10^{29} erg s^{-1}. Simple calculations show, that most of this energy will be emitted in X-rays with a typical blackbody temperature of about 0.1 keV (if due to significant magnetic field accretion proceeds onto small polar caps then the temperature would be higher up to 1 keV). Note, that the Bondi rate is just the upper limit, in reality due to heating (Shvartsman 1970a, Shvartsman 1971, Blaes et al. 1995) and magnetospheric effects (Toropina et al. 2001) the accretion rate can be lower.

Due to the turbulence the accreted matter carries non-zero angular momentum:

$$j_t = v_t(R_G) \cdot R_G = v_t(R_t) R_t^{-1/3} R_G^{4/3} .$$

In this formula it is considered that cells of the size $r = R_G$ are the most important, otherwise in the Eq.(1) below it is necessary to introduce factor $\alpha \neq 1$.

If j_t is larger than the Keplerian value at the magnetosphere boundary (i.e. at the Alfven radius $R_A = (\mu^2/2\dot{M}\sqrt{GM})^{2/7}$, here μ is the magnetic moment of a NS) an accretion disk is formed around the AINS. In the disk a part of the angular momentum is carried outwards, and the NS accretes matter with Keplerian angular momentum $j_K = v_K(R_A) \cdot R_A$, v_K – Keplerian velocity. This situation holds only for very low magnetic fields and low spatial velocities of NSs, thus we do not take it into account in the present calculations.

We consider the lowest spatial velocity of NSs[2] to be equal to 10 km s^{-1}. This value is of order of the speed of sound. Therefore lower spatial velocities can not change the accretion rate significantly. Moreover such low values are not very probable due to non-zero spatial velocities of NSs progenitors (see Popov et al. 2000a, Arzoumanian et al. 2001 for limits onto the fraction of low velocity INSs derived by different methods). For several cases below we consider $v \simeq v_{NS}$.

During the time required to cross a turbulent cell of the size R_G, $\Delta t = R_G/v_{NS} \simeq 11.8\,(v/10\,\mathrm{km\ s^{-1}})^{-3}$ yrs, the change in the angular momentum of a NS J is given as:

$$|\boldsymbol{\Delta J}| = \dot{M} j_t \Delta t = \pi n m_p v_t(R_t) R_t^{-1/3} R_G^{13/3}$$
$$\simeq 1.23 \cdot 10^{39}\,\mathrm{g\,cm^2\,s^{-1}} \times$$
$$n \left(\frac{v_t(R_t)}{10\,\mathrm{km\,s^{-1}}}\right) \left(\frac{R_t}{2 \cdot 10^{20}\,\mathrm{cm}}\right)^{-1/3} \left(\frac{v}{10\,\mathrm{km\,s^{-1}}}\right)^{-26/3} .$$

Accordingly, the change of the spin frequency reads:

$$|\boldsymbol{\Delta\omega}| = |\boldsymbol{\Delta J}|/I$$

[1] Any other more complicated model of turbulence requires better knowledge of the ISM structure especially on small scales. This information can be obtained from radio pulsar scintillation observations (see for example Smirnova et al. 1998). We plan to include this data in our future calculations.

[2] The lowest measured velocity is $\gtrsim$ 50 km s^{-1} (Lyne & Lorimer 1994), please bear in mind that selection effects are very important here.

$$\simeq 1.23 \cdot 10^{-6}\,\mathrm{s}^{-1} \times$$
$$n \left(\frac{v_t(R_t)}{10\,\mathrm{km\,s^{-1}}}\right) \left(\frac{R_t}{2\cdot 10^{20}\,\mathrm{cm}}\right)^{-1/3} \left(\frac{v}{10\,\mathrm{km\,s^{-1}}}\right)^{-26/3} I_{45}^{-1}\,,$$

here $I = I_{45} 10^{45}$ g cm^2 — moment of inertia of a NS. The orientation of $\boldsymbol{\Delta\omega}$ is random, and it is isotropically distributed on a sphere. The value of the frequency change is strongly dependent on the spatial velocity of the NS: $\Delta\omega \sim v^{-26/3}$, so the maximum value for $v_{NS} = 10$ km s^{-1} is $\Delta\omega_{\max} \simeq 6 \cdot 10^{-8}$ rad s^{-1}.

On this view, it follows that it is possible to describe the spin evolution of an AINS as random movement in 3-D space of angular velocities $\boldsymbol{\omega}$. Since typical temporal and "spatial" scales Δt and $\Delta\omega$ are reasonably small ($\Delta t \ll t_{gal} \simeq 10^{10}$ yrs, $\Delta\omega \ll \omega \sim 10^{-1} \div 10^{-7}$ rad s^{-1}) we consider the problem to be continuous. Therefore, we can use differential equations valid for continuous processes.

In this case the spin evolution of an AINS is described by the diffusion equation with the coefficient D given by:

$$D = \frac{\alpha}{6}\frac{\Delta\omega^2}{\Delta t}, \qquad (1)$$

here α is a coefficient which takes into account cells with sizes $r \neq R_G$ (everywhere in this paper we use $\alpha = 1$).

Apart from the influence of random turbulence, an AINS is spinning down due to magnetic braking:

$$\frac{d\boldsymbol{\omega}}{dt} = \kappa_t \frac{\mu^2}{I R_{co}^3}\frac{\boldsymbol{\omega}}{|\omega|} + \boldsymbol{F}_t = \kappa_t \frac{\mu^2}{IGM}\omega^2 \frac{\boldsymbol{\omega}}{|\omega|} + \boldsymbol{F}_t\,. \qquad (2)$$

Here $R_{co} = (GM/\omega^2)^{1/3}$ is the corotation radius, κ_t is a constant of order unity, which takes into account details of the interaction between the magnetosphere and the accreted matter (everywhere in this paper we assume $\kappa_t = 1$), $\boldsymbol{F}_t$ is the random (turbulent) force, which has a zero average value: $\langle \boldsymbol{F}_t \rangle = 0$.

On large time scales and for relatively short spin periods we can neglect the momentum of the accreted matter and the initial period, p_0, of the NS. In that case the solution of Eq. (2) looks as:

$$\omega \propto t^{-1}\,, \quad p \equiv \frac{2\pi}{\omega} \propto t\,.$$

Magnetic braking produces a convective term in the evolutionary equation for the spin frequency distribution, $f(\omega)$. As far as the initial distribution of spin vectors is isotropic (magnetic braking does not change the orientation of $\boldsymbol{\omega}$) and the turbulent diffusion is isotropic too, we obtain a spherically symmetric distribution in space of the angular velocities, i.e.:

$$f(\boldsymbol{\omega}) = f_3(\omega)$$

($f_3(\omega)$ describes the 3-D distribution of AINSs, its dimension is [s^{-3}]).

From (1) and (2) we obtain the evolutionary equation:

$$\frac{\partial f_3}{\partial t} = \frac{A}{\omega^2}\frac{\partial}{\partial\omega}\left(\omega^4 f_3\right) + \frac{D}{\omega^2}\frac{\partial}{\partial\omega}\left(\omega^3 \frac{\partial f_3}{\partial\omega}\right)\,, \qquad (3)$$

here $A = (\mu^2/IGM)$.

The boundary condition at $\omega = 0$ can be derived from the equality of the flow of the particles, which at this point becomes zero:

$$\left.\frac{\partial f_3}{\partial\omega}\right|_{\omega=0} = 0\,.$$

It is easy to find a stationary solution of equation (3) on the semi-infinity axis[3] ($\omega \geq 0$)

$$f_3^{\mathrm{st}} = C_{st}\exp\left(-\frac{\mu^2}{3\,IGMD}|\omega|^3\right)\,, \qquad (4)$$

here the normalization constant C_{st} can be derived from the condition $4\pi\int f_3^{\mathrm{st}}(\omega)\omega^2 d\omega = N$, N is the total number of AINSs in the distribution. The total number of NSs in the Galaxy is uncertain, $N \sim 10^8$–10^9. Population synthesis calculations (Popov et al. 2000b) give arguments for a higher total number about 10^9. Here we do not address this question. From the curves below, which represent *relative* period distribution of AINSs, one can determine absolute numbers of AINSs with each period value if the total number of these objects is known.

The position of the maximum of $f_3^{\mathrm{st}}(p)$ depends on v, μ, n and M. An increase of v and μ shifts the maximum to larger p, an increase of n and M shifts the maximum to shorter p.

If we have to solve the problem for a constant starformation rate we can write the second boundary condition as:

$$f_3(\omega_{\mathrm{A}}) = \frac{IGM}{4\pi\kappa_t\mu^2}\cdot\frac{\dot N}{\omega_{\mathrm{A}}^4}\,,$$

here $\dot N$ is the rate at which NSs come to the stage of accretion, $\omega_{\mathrm{A}} = 2\pi/p_A$ is the *accretor* frequency, at that value for given v and n accretion sets on, $p_A = 2^{5/14}\pi(GM)^{-5/7}(\mu^2/\dot M)^{3/7} \simeq 300\,\mu_{30}^{6/7}(v/10\,\mathrm{km\,s^{-1}})^{9/7} n^{-3/7}$ s.

The function $f_3(\omega)$ is connected with distributions of the absolute values of the angular velocity, $f_1(\omega)$, and of the spin period, $f(p)$, according to the formulae:

$$f_1(\omega) = 4\pi\omega^2 f_3(\omega)\,,$$

$$f(p) = \frac{32\pi}{p^4} f_3\left(\frac{2\pi}{p}\right)\,.$$

Figure 1 shows the evolution of $f(p)$ as a function of time for typical parameters of an AINS and the ISM.

For given v, μ and n NSs enter the *accretor* stage when they spin down to p_{A}. Up to this value they evolve as *ejectors* and *propellers* (Lipunov 1992, Colpi et al. 2001).

For $p_{\mathrm{A}} < p < p_{cr}$ the influence of the turbulence is small and AINS spin down according to $p \propto t$. Here p_{cr} defines the stage when spin changes due to magnetic braking and turbulent acceleration/deceleration become of the same order of magnitude (see below the Discussion). One

[3] As far as NS enter the stage of accretion with relatively short periods $\omega_{\mathrm{A}} \gg \langle\omega(f_3^{\mathrm{st}})\rangle$, the obtained distribution is differed from the real one only on high frequencies $\omega \simeq \omega_{\mathrm{A}}$.

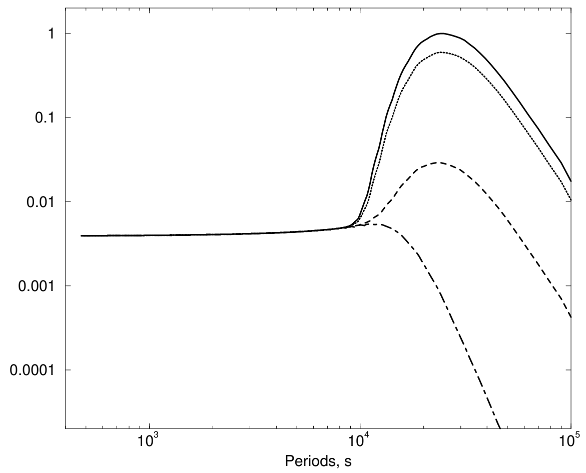

Fig. 1. Evolution of the period distribution in time; $\mu = 10^{30}$ G cm^3, $n = 1$ cm^{-3}, $v_{NS} = 10$ km s^{-1}. Curves are plotted for 4 different moments from $1.72 \cdot 10^9$ yrs to $9.8 \cdot 10^9$ yrs (t_A for the chosen parameters is equal to $\simeq 1.7 \cdot 10^9$ yrs). An AINS crosses the horizontal part from $\simeq 10^2$ s to 10^4 s in $\sim 6 \cdot 10^7$ yrs. Curves were normalized to 1 in the maximum for the highest curve.

can introduce another useful timescale, Δt_{cr}. This is the time required to reach turbulent regime, $p = p_{cr}$. These considerations together with constant starformation rate determine left part of the distribution where $f(p) = const$ (see Fig. 1).

As the period grows turbulence becomes more and more important. Finally the first AINSs reach the point $p = \infty$ ($\omega = 0$) and there the equilibrium component of the distribution (described by Eq. (4) is formed quickly. This component decreases in a power law fashion ($f(p) \propto p^{-4}$) at $p \gg p_{turb}$ (where p_{turb} is determined by the width of the distribution (4)), and decreases even faster than exponentially at $p \ll p_{turb}$. Please note that the number of AINSs reaching equilibrium grows linearly with time. As a result, the amplitude of the equilibrium component of the distribution grows likewise. If the time required to reach the *accretor* stage, t_A, is large ($t_{gal} - t_{\rm A} \ll t_{gal}$) the equilibrium component is not formed. For $t_{\rm A} > t_{gal}$ NS never reach the stage of accretion.

Similar considerations can be applied also to binary X-ray pulsars (see Lipunov 1992). In binaries NSs can reach a real period equilibrium (Ghosh & Lamb 1979), and stationary solution should be used. In that case observations of period fluctuations can help to derive physical parameters of the systems, for example stellar wind velocity for wind-fed pulsars (Lipunov & Popov 1995b).

3. Calculations and results

The main aim of this paper is to obtain period distribution for an AINS in a turbulent ISM. We assume that NS are born with short ($\ll 1$ s) spin periods. The magnetic moment distribution is taken to be in log-Gaussian form:

$$f(\mu) = \frac{1}{\sqrt{2\pi\sigma_m}} \exp\left(\frac{(\log\mu - \log\mu_0)^2}{2\sigma_m^2}\right), \qquad (5)$$

with $\log\mu_0 = 30.06$ and $\sigma_m = 0.32$ (see Colpi et al. 2001 and references therein for a recent discussion and data on magnetism in NSs). The magnetic field is considered to be constant (see, for example, Urpin et al. 1996 for calculations of magnetic field decay and related discussion).

The velocity distribution (due to kick after supernova explosion) is assumed to be Maxwellian:

$$f(v_{NS}) = \frac{6}{\sqrt{\pi}} \frac{v_{NS}^2}{v_m^2} \exp\left(\frac{3}{2}\frac{v_{NS}^2}{v_m^2}\right) \qquad (6)$$

with $v_m = 200$ km s^{-1} which corresponds to $\sigma_v \simeq 140$ km s^{-1} (see for example Lyne & Lorimer 1994, Cordes & Chernoff 1997, Hansen & Phinney 1997, Lorimer et al. 1997, Cordes & Chernoff 1998, and Arzoumanian et al. 2001 for discussions on the kick velocity of NSs).

We use two values of the ISM number density, $n = 1$ cm^{-3} and $n = 0.1$ cm^{-3}, as typical values for the Galactic disk [4].

All NS are divided into 30 groups in μ interval from $10^{28.6}$ to $10^{31.6}$ G$\cdot$cm^3 with the step 0.1 dec and 49 groups in velocities ranging from 10 to 500 km s^{-1} with a step of 10 km s^{-1}.

For each group we calculate t_A, the time necessary to reach accretion:

$$t_A = t_E + t_P,$$

here t_E and t_P — durations of the *ejector* and *propeller* stages correspondently.

In the stage of ejection a NS spins down due to magneto-dipole losses:

$$t_E \simeq 10^9 n^{-1/2} \left(\frac{v}{10\ \mathrm{km\,s^{-1}}}\right) \left(\frac{\mu}{10^{30}\ \mathrm{G \cdot cm^3}}\right)^{-1} \mathrm{yrs.}$$

The spin down in the *propeller* stage (Shvartsman 1970b, Illarionov & Sunyaev 1975) is not well known, but for a constant field always $t_P < t_E$ (Lipunov & Popov 1995a) (see also results of numerical calculations in Toropin et al. 1999), so we neglect t_P in the rest of the paper, i.e. we assume $t_A = t_E$.

For high spatial velocities after the *ejector* phase an INS appears not as a *propeller*, but as a so-called *georotator* (Lipunov 1992, see also Rutledge 2001). At this stage $R_A > R_G$, so the surrounding matter does not feel the gravitation of the NS, and its magnetosphere looks like magnetospheres of the Earth, Jupiter etc. Partly because of that effect we truncated our velocity distribution at 500 km s^{-1}.

For systems with $t_A < t_{gal} = 10^{10}$ yrs we solve Eq. (3) on the time interval $t_{gal} - t_A$. For angular velocities

[4] This assumption is reasonable for relatively low velocity INS, but only this fraction of the whole population is important for us here.

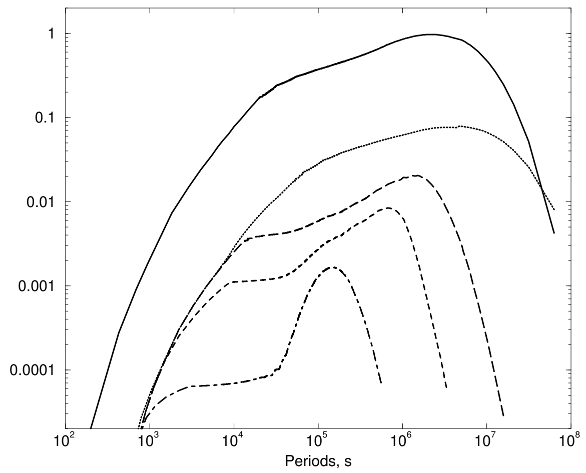

Fig. 2. The period distribution for populations of AINSs evolving in an ISM with number density $n = 1$ cm^{-3} (upper solid curve) and $n = 0.1$ cm^{-3} (the second dotted curve). Lower curves show results for the low velocity part of the AINS population for $n = 0.1$ cm^{-3} ($v < 60$, 30, 15 km s^{-1} correspondently, see Discussion below). Curves are normalized to 1 at the maximum of the highest curve.

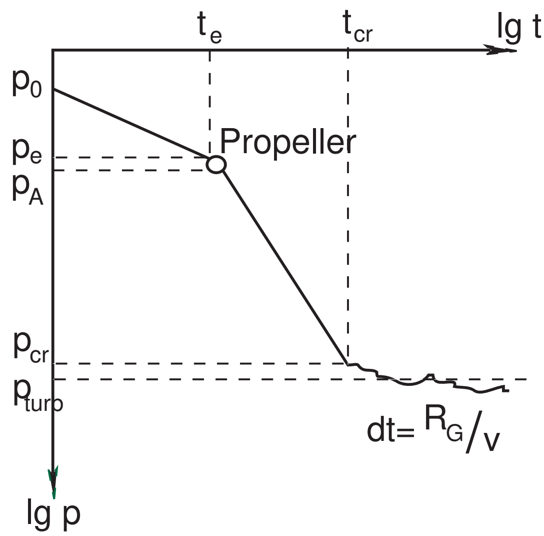

Fig. 3. Schematic view of the period evolution of an INS. A NS starts with $p = p_0$, then spins down according to magneto-dipole formula up to p_E, then a short *propeller* stage (marked by a circle) appears and lasts down to $p = p_A$. At the situation illustrated in the figure $t_E \approx t_A$.
On the stage of accretion at first magnetic braking is more important, after at $t = t_{cr}$ turbulence starts to influence significantly spin evolution of a NS period stars to fluctuate with a typical time scale R_G/v_{NS} close to mean value noted p_{turb} on the vertical axis.

we use grid with 200 cells from $\omega = 0$ to ω_A. A conservative implicit scheme is used. The derived distributions $f(p, \mu, v)$ (for different groups) are summed together with weights from Eqs. (5) and (6).

The final distribution for $n = 1$ cm^{-3} and $n = 0.1$ cm^{-3} are shown on Fig. 2.

We note, that the final distribution significantly differs from the one for a single set of parameters (n, μ, v) as can be seen from Fig. 1 and Fig. 2. In the final one we a see power-law cutoff ($f \propto p^{-4}$) at long periods ($p > 10^7$ s) similar to the cutoff in the Fig. 1. Then at the intermediate periods ($10^4 < p < 10^7$ s) the curve has a power-law ascent due to summation of individual curve maxima. Finally a sharp cutoff at short periods ($p < 10^4$ s) is present because different INSs enter the *accretor* stage with different periods: $p_A = p_A(n, \mu, v)$.

4. Discussion and conclusions

After an INS comes to the stage of accretion ($p > p_A$) its spin is controlled by two processes (see Eq.2): magnetic spin-down and turbulent spin-up/spin-down. A schematic view of such evolution process is shown in Fig. 3.

We can describe them with characteristic timescales: t_{mag} and t_{turb}. As one can calculate, $t_{mag} \sim p$ and $t_{turb} \sim p^{-1}$. Here we evaluate t as $p/\dot{p}$, and $\dot{p}_{turb} = p^2 \dot{M} j/(2\pi I)$. Here we again note, that j_t can be larger that the Keplerian value of angular momentum at the magnetosphere, $j_K = v_K(R_A), R_A$. So we write j instead of j_t as we did before, now $j = \min(j_K, j_t)$.

$$\dot{p}_{mag} = 2\pi\mu^2/(IGM) \tag{7}$$

Initially (immediately after $p = p_A$) magnetic spin-down is more significant:

$$t_{mag} = \frac{IGM}{2\pi\mu^2} p, \tag{8}$$

but at some period, p_{cr}, these two timescales should become of the same order, and for longer periods an INS will be governed mainly by turbulent forces.

One can obtain the following formula:

$$p_{cr}^2 = \frac{4\pi^2\mu^2}{GM\dot{M}j}. \tag{9}$$

This period lies between p_A and p_{turb}. An INS reaches p_{cr} in $\Delta t_{cr} \sim 10^5$–10^7 yrs after onset of accretion:

$$\Delta t_{cr} = \frac{I\sqrt{GM}}{\mu\sqrt{\dot{M}j}}. \tag{10}$$

Here Δt_{cr} is calculated approximately as $p_{cr}/\dot{p}_{mag}$.

The period evolution of an AINS can be described in the following way (Fig. 3): after the INS comes to the stage of accretion it spins down for $\sim 10^5 - 10^7$ yrs up to p_{cr}, then the evolution is mainly controlled by the turbulence, and period fluctuates with typical value p_{turb}, which is determined by the properties of the surrounding ISM and spatial velocity of an INS.

We note, that we do not take into account any selection effects. For example, as period and luminosity both depend on the velocity of an INS, they are correlated. Taking into account the lower flux limits attainable by the present day satellites it is possible to calculate the probability to *observe* an accreting INS with some period. Also periods (and luminosities) are correlated with the position of an INS in the Galaxy.

So, it is reasonable to make calculations which include all these effects in order to make better predictions for observations. We plan to unite our population synthesis calculations with detailed calculations of spin evolution later. In this paper we present distributions *as if* all accreting INSs can be observed.

For the main part of its life the spin period of a low-velocity AINS is governed by turbulent forces. The characteristic timescale for them can be written as: $t_{turb} = I\omega/(\dot{M}j)$, and it is equal to 10^4-10^5 yrs for typical parameters.

If the field decays the picture should become completely different (see for example Konenkov & Popov 1997, Wang 1997), and observations of AINSs can put important limits onto models of magnetic field decay in NS (Popov & Prokhorov 2000). For decaying fields AINSs can appear as pulsating sources with periods about 10 s and $\dot{p}$ about 10^{-13} s/s (Popov & Konenkov 1998). The value and sign of $\dot{p}$ will fluctuate as an INS passes through the turbulent cell on a time scale R_G/v_{NS}, which is about a year for typical parameters. Irregular fluctuations of $\dot{p}$ on that time scale can be significant indications for the accretion (vs. cooling) nature of observed luminosity.

Roughly $\dot{p}$ can be estimated from the expression:

$$|\dot{p}| \le p^2 \frac{\dot{M}j}{2\pi I} \sim v^{-17/3}. \tag{11}$$

Here we neglect magnetic braking in Eq. (2). This equation for $\dot{p}$ is valid for the turbulent regime of spin evolution, i.e. for $p \gg p_{turb}$. For a given value of the spin period $\dot{p}$ fluctuates between $+p^2\frac{\dot{M}j}{2\pi I}$ and $-p^2\frac{\dot{M}j}{2\pi I}$, and depends on n, v, but not on μ (if $j = j_t$, not $j = j_A$). In this picture the $\dot{p}$ distribution in the interval specified above (Eq.11) is flat.

The behavior of p and $\dot{p}$ of AINSs in molecular clouds can be different (Colpi et al. 1993), especially for low spatial velocities of NSs. Some of our assumptions in that case are not valid, and the results cannot be applied directly. But we note, that passages through molecular clouds are relatively rare and short, they cannot significantly influence the general picture of AINSs spin evolution.

The period distribution which can be obtained from observation (for example from $ROSAT$ data) can be different from the two upper curves in Fig. 2. Such surveys are flux-limited, so they include (in the case of AINS) the most luminous objects. But they form only a small fraction of the whole population.

To illustrate this in Fig. 2 we also plot distributions for low velocity objects ($v < 60, 30, 15$ km s^{-1}). In the case of fixed a ISM density an upper limit to the value of the space velocity corresponds to a lower limit to the accretion luminosity of an AINS. Clearly, the brighter the source, the shorter (on average) its spin period. Even in such groups with relatively short periods their values are far from typical periods of $ROSAT$ INSs, $\sim 5 - 20$ s. So, these objects cannot be explained by *accretors* with constant field $B \sim 10^{12}$ G.

Calculations of period distributions for decaying magnetic field, for populations of INSs with a significant fraction of magnetars and for an accretion rate different from the standard Bondi-Hoyle-Littleton value (due to heating and influence of a magnetosphere) will be done in a separate paper.

In conclusion we stress readers attention on the main results of the paper:

— we obtained spin period distributions for AINSs with constant magnetic field and "pulsar" properties (magnetic fields, initial periods and velocity distributions).
— these distributions are shown in Fig. 2. They have a broad maximum at very long periods, $\sim \mathbf{10^5} - \mathbf{10^6}$ s. In that case the observed objects should not show any periodicity.
— the periods of these objects should fluctuate on a time scale $R_G/v_{NS} \sim 1$ yr.

Acknowledgements. This work was supported by grants of the RFBR 01-02-06265, 00-02-17164, 01-15(02)-99310.

AK thanks Sternberg Astronomical Institute for hospitality. SP and MP thank Monica Colpi, Roberto Turolla and Aldo Treves for discussions and Universities of Como, Milano and Padova for hospitality.

We thank Vasily Belokurov and the referee of the paper for their comments on the text and useful suggestions.

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