SPIN AND MAGNETISM IN OLD NEUTRON STARS

Monica Colpi[1], Andrea Possenti[2], Serge Popov[3], and Fabio Pizzolato[4]

[1] Università di Milano Bicocca, Dip. di Fisica, P.zza della Scienza 3, Milano
[2] Osservatorio Astronomico di Bologna, Via Ranzani 1, 40127 Bologna
[3] Sternberg Astronomical Institute, Universitetskii Pr. 13, 119899, Moscow
[4] Istituto di Fisica Cosmica G. Occhialini, Via Bassini 15, 20133 Milano

Abstract. The thermal, spin and magnetic evolution of neutron stars in the old low mass binaries is first explored. Recycled to very short periods via accretion torques, the neutron stars lose their magnetism progressively. If accretion proceeds undisturbed for 100 Myrs these stars can rotate close to break up with periods far below the minimum observed of 1.558 ms. We investigate their histories using population synthesis models to show that a tail should exist in the period distribution below 1.558 ms. The search of these ultrafastly spinning neutron stars as pulsars can help discriminating among the various equations of state for nuclear matter, and can shed light into the physics of binary evolution.

The evolution of isolated neutron stars in the Galaxy is explored beyond the pulsar phase. Moving through the tenuous interstellar medium, these old solitary neutron stars lose their rotational energy. Whether also their magnetism fades is still a mystery. A population synthesis model has revealed that only a tiny fraction of them is able to accrete from the interstellar medium, shining in the X-rays. There is the hope that these solitary stars will eventually appear as faint sources in the *Chandra* sky survey. This might give insight on the long term evolution of the magnetic field in isolated objects.

1 Introduction

The amount of rotation and the strength of the magnetic field determine many of the neutron star's observational properties. Over the neutron star lifetime, the spin and the field change and the study of their evolution provides important clues into the physics of the stellar interior.

Of the billion neutron stars in the Galaxy, we shall be mainly concerned with the evolution of two distinct populations: The millisecond pulsars and the isolated neutron stars both aging, the first in low mass binaries, the second as field stars moving in the interstellar medium of our Milky Way. We will show that the interaction with their surroundings may profoundly alter their spin and magnetic field. The extent of these changes and the modes vary in the two scenarios. It is in exploring this diversity that we wish to infer the nature of the equation of state and to provide a unified view of field decay. In particular, the existence of "unconventional" sources, such as *sub-millisecond pulsars*, stars rotating close to their break up limit, and as *solitary neutron stars accreting* *the interstellar medium,* is a crucial test for our studies. Their discovery is an observational and theoretical challenge.

In Section 2 we trace the evolution of a neutron star in the Ejector, Propeller and Accretion phases, described briefly using simple background arguments. Section 3 surveys the physical models for the evolution of the magnetic field both in isolated and accreting systems. In Section 4 we explore possible individual pathways that may lead to the formation, in low mass binaries, of neutron stars spinning very close to their mass shedding threshold, a limit sensitive to the equation of state for nuclear matter. Pathways of isolated neutron stars follow. The first four sections set the general background used to construct, in Sections 5 and 6, statistical models aimed at determining the presence and abundance of sub-millisecond pulsars in the Galaxy, and of solitary neutron stars shining in the X-rays. Specifically, in Section 5 we explore, within the recycling scenario, the star's spin and magnetic evolution using physical models and the role played by disc instabilities in affecting the latest phases of binary evolution toward the Ejector state of sub-millisecond pulsars. In Section 6 we carry out the first stellar census. We then establish how elusive neutron stars can be as accreting sources from the interstellar medium due to their large velocities and to magnetic field decay.

2 The Ejector, Propeller and Accretion Phases

Over the stellar lifetime, magnetic and hydrodynamic torques acting on the neutron star (NS hereafter) induce secular changes in its spin rate. Four physical parameters determine the extent of the torques: the magnetic field strength B, the rotational period P, the density of the surrounding (interstellar) medium n, and the rate of mass inflow $\dot{M}$ toward the NS. According to the magnitude of these quantities, a NS experiences three different evolutionary paths: Ejector ($\mathcal{E}$), Propeller ($\mathcal{P}$), and Accretion ($\mathcal{A}$) [1]. In phase $\mathcal{E}$ the NS braking results from the loss of magneto-dipole radiation as in an ordinary pulsar. The implied spin-down rate is

$$\frac{dP}{dt} = \frac{2\pi^2}{c^3 I}\frac{\mu^2}{P} \quad , \tag{1}$$

where I is the NS moment of inertia and $\mu = B_{\rm s}R_{\rm s}^3/2$ the NS magnetic moment, function of the stellar radius $R_{\rm s}$ and polar magnetic field $B_{\rm s}$. The torque decreases with increasing P, and at the current period P (much longer than the initial period) the time spent in phase $\mathcal{E}$ is

$$\tau_{\mathcal{E}} = \frac{c^3\, I\, P^2}{4\pi^2\, \mu^2}. \tag{2}$$

Resulting from the emission of electromagnetic waves and charged particles, the rotational energy loss of equation (1) can proceed well beyond the active radiopulsar phase and the magneto-dipolar outflow creates a hollow cavern nesting the NS. Phase $\mathcal{E}$ remains active as long as the characteristic radius $r_{\rm st}$ of this cavern (the *stopping radius* [1] [2]) is larger than both the *light cylinder* radius

$$r_{\rm lc} \;=\; \frac{cP}{2\pi} \tag{3}$$

and the *gravitational radius*

$$r_{\rm G} \;=\; \frac{2GM}{c_s^2 + v_{\rm rel}^2} \tag{4}$$

where c_s is the sound speed of the interstellar medium (ISM) and $v_{\rm rel}$ the NS velocity relative to the medium. Phase $\mathcal{E}$ ends when matter leaks through $r_{\rm st}$, and this occurs when the period exceeds

$$P_{\mathcal{E}\to\mathcal{P}} = \frac{2\pi^{3/4}}{c\sqrt{2GM}}\mu^{1/2}v^{1/2}\varrho^{-1/4} \tag{5}$$

where $v = (c_s^2 + v_{\rm rel}^2)^{1/2}$ and ϱ is the mass density of the ISM [3]. At $P_{\mathcal{E}\to\mathcal{P}}$ the outflowing pulsar momentum flux is unable to balance the ram pressure of the gravitationally captured matter at $r_{\rm G}$ [3]. Matter initiates its infall and approaches the *magnetospheric radius*

$$r_{\rm A} \;=\; \left(\frac{\mu^4}{2GM\dot{M}^2}\right)^{1/7}. \tag{6}$$

Here the magnetic pressure of the dipolar stellar field ($\propto r^{-6}$) balances the ram pressure of the infalling material ($\propto r^{-5/2}$), accreting at a rate $\dot{M}$. At $r_{\rm A}$ the steeply rising magnetic field would thread the flow, enforcing it to corotation. However, not always can matter trespass this edge: penetration is prevented whenever the rotational speed Ω of the uniformly rotating magnetosphere exceeds the local Keplerian velocity ω_K $(= [GM/r_{\rm A}^3]^{1/2})$ at $r_{\rm A}$. This condition translates into a comparison between the *corotation radius*

$$r_{\rm cor} \;=\; \left(\frac{GM}{\Omega^2}\right)^{1/3} \tag{7}$$

and $r_{\rm A}$. When $r_{\rm cor}$ lies inside $r_{\rm A}$ the magnetosphere centrifugally lifts the plasma above its escape velocity, inhibiting its further infall toward the NS surface: this is the propeller phase $\mathcal{P}$. The magnetically driven torques lead to a secular spin-down of the NS occurring at a rate

$$\frac{d}{dt}(I\Omega) \;=\; \xi\dot{M}r_{\rm cor}^2\,[\omega_{\rm K}(r_{\rm A}) \;-\; \Omega]\,, \tag{8}$$

where $\dot{M}$ is the accretion rate at $r_{\rm A}$, and ξ a numerical factor dependent on the accretion pattern, ranging from $\xi \simeq 1$ for disc accretion to $\xi \simeq 10^{-2}$ for spherical accretion. The effectiveness of the propeller in spinning down the NS and in ejecting matter far out from the magnetosphere is still largely model dependent [4]. Recently, evidence that this mechanism is at work in binaries of high [5] and low mass [6] has come from X-ray observations.

Phase $\mathcal{P}$ terminates only when the corotation radius $r_{\rm cor}$ increases above $r_{\rm A}$; thereon the NS is able to accept matter directly onto its surface: this is the accretion phase $\mathcal{A}$. The transition $\mathcal{P}\to\mathcal{A}$ occurs at a critical NS spin rate $P_{\mathcal{P}\to\mathcal{A}}$ obtained by equating $r_{\rm A}$ and $r_{\rm cor}$:

$$P_{\mathcal{P}\to\mathcal{A}} = \frac{2\pi}{\sqrt{GM}}\left(\frac{\mu^4}{2GM\dot M^2}\right)^{3/14} \propto \frac{B^{6/7}}{\dot M^{3/7}}. \tag{9}$$

In isolated NS, the accretion flow is almost spherically symmetric and the NS period P, after the onset of $\mathcal{A}$, may vary erratically due to the positive and negative random torques resulting from the turbulent ISM [7] [8] [1]

In a disc-like geometry (in binaries) the spin evolution during accretion is guided by the advective, magnetic and viscous torques present in the disc. Depending on their relative magnitude, these torques can either spin the NS up or down. The rate of change of the NS angular momentum is obtained integrating the different contributions over the whole disc, yielding

$$\frac{d}{dt}(I\Omega) \simeq \dot M r_{\rm A}^2\omega_{\rm K}(r_{\rm A}) + \int_{r_{\rm A}}^{\infty} B_z^2\frac{R^3h}{\eta}[\omega_{\rm K}(R)-\Omega]dR \ , \tag{10}$$

where η is the plasma electrical diffusivity, h the disc scale thickness and R the radial cylindrical coordinate (in eq. 10 we neglect the local viscous contribution as it vanishes at $r_{\rm A}$). The first term in the rhs of equation (10) describes the advection of angular momentum by accretion. Magnetic torques are instead non-local, resulting from an integral that extends over the whole disc: different regions can give either positive (spin-up) or negative contributions (spin-down). When $\Omega \gg \omega_K(r_{\rm A})$, the integrand in (10) is negative yielding a spin-down magnetic torque. On the other hand, if the star rotates very slowly ($\Omega \ll \omega_K(r_{\rm A})$) the overall magnetic torque is positive, leading to a secular spin-up: when the total torque is positive the spin-up process is termed *recycling*. At a critical (intermediate) value of Ω, termed *equilibrium angular velocity* $\omega_{\rm eq}$, the magnetic torque is negative, and its value exactly offsets the (positive) advective torque. In this regime the NS accretes matter from the disc without exchanging angular momentum at all. The precise value of $\omega_{\rm eq}$ is rather uncertain, but it ought to lie between $0.71-0.95$ times the critical frequency $\omega_{\rm cr}=2\pi/P_{\mathcal{P}\to\mathcal{A}}$, in the mid of the propeller and accretor regimes (see [9] and references therein). Since $\omega_{\rm cr}$ and $\omega_{\rm eq}$ have near values, they are often confused in the literature. There is a narrow range for Ω between $\omega_{\rm eq}$ and $\omega_{\rm cr}=2\pi/P_{\mathcal{P}\to\mathcal{A}}$ within which the NS accretes mass but spins down.

Once the star has reached its equilibrium period $P_{\rm eq}=2\pi/\omega_{\rm eq}$, further changes in the spin period P are possible only if the magnetic field and/or

[1] Old accreting isolated NSs are thus expected to show strong fluctuations of $\dot P$ over a time scale comparable to the crossing time $\tau_{\rm fluct}\sim min(r_{\rm ISM}, r_G)/v_{\rm rel}$, where $r_{\rm ISM}$ is the spatial scale of the ISM inhomogeneities.

$\dot{M}$ vary. Generally, field decay causes the star to slide along the so called *spin-up line* ($P = P_{\rm eq}(\mu, \dot{M})$) at the corresponding accretion rate $\dot{M}$. The spin period decreases on the time scale τ_μ, as $P_{\rm eq}$ is proportional to $\mu^{6/7}$. A large increase in the mass transfer rate can induce further spin up since the magnetosphere at $r_{\rm A}$ is squeezed in a region of higher Keplerian velocity. A decrease in $\dot{M}$ has the opposite effect and if there is a decline in the mass transfer rate (due to disc instabilities and/or perturbations in the atmosphere of the donor star), the NS can transit from $\mathcal{A} \to \mathcal{P} \to \mathcal{E}$. The critical period for the last transition does not coincides with equation (5), but occurs when the Alfven radius $r_{\rm A}$ (function of the mass transfer rate $\dot{M}$) exceeds the light cylinder radius $r_{\rm lc}$ (eqs. 3 and 6), i.e., when

$$P_{\mathcal{P}\to\mathcal{E}} = \frac{2\pi}{c}\left(\frac{\mu^4}{2GM\dot{M}^2}\right)^{1/7}, \tag{11}$$

(note that this transition is not symmetric relative to $\mathcal{E} \to \mathcal{P}$). Over the stellar lifetime, NSs can trace loops moving through the various phases ($\mathcal{P} \leftrightarrow \mathcal{A}$ or/and $\mathcal{P} \leftrightarrow \mathcal{E}$). It may happen that $r_{\rm A}$ increases above $r_{\rm G}$ when $\dot{M}$ is very low (as in the ISM): in this case the star is in the so called Georotator state ($\mathcal{G}$).

The major drivers of the NS evolution are magnetic field decay (though amplification is an interesting possibility) and variations in the mass transfer rate: their study is thus the subject of §3 and §4..

3 Magnetic field evolution

In the μ versus P diagram of pulsars (PSRs) we clearly find NSs endowed by intense magnetic fields (the canonical pulsars) and the "millisecond" pulsars (MSPs), NSs with a much lower magnetic moment. The first are isolated objects and largely outnumber the millisecond pulsars often living in binaries with degenerate companion stars. The origin of this observational dichotomy seems understood but the physical mechanisms driving the field evolution remain still uncertain. We review here current ideas for the evolution of the B field. They are applied later when investigating (i) the existence of sub-millisecond pulsars, and (ii) the nature of the six isolated NSs discovered by *Rosat*.

3.1 Historical and observational outline

Soon after the discovery of PSRs, Ostriker & Gunn [10] proposed that their surface magnetic field should not remain constant but decay. Under this assumption they explained the absence of PSRs with periods much longer than one second (the decay of μ would bring the objects below the death line, turning off the active radio emission [11]) and the claimed dimming of the radio luminosity L_R proportional to μ^2 [12]. Their pioneering statistical study suggested that the NSs at birth have very high magnetic moments $\mu \simeq 10^{29} \div 10^{31}$ G cm^3, whose values decay exponentially due to ohmic dissipation in the NS interior, on a typical time-scale [13]

$$\tau_\mu = 4\sigma R^2/\pi c^2 \ , \tag{12}$$

where σ is the conductivity, taken as uniform. Ostriker & Gunn evaluated τ_μ using the electrical conductivity for the crystallized crust ($\simeq 0.6 \times 10^{23}$ sec^{-1} [14]), thereby assuming implicitly that the magnetic field resides in the outer layers ($\rho < 10^{14}$ g cm^{-3}) of the star, and obtained $\tau_\mu \sim 4 \times 10^6$ yr. But in the same year, Baym, Pethick & Pines [15] argued that the magnetic field pervades the entire star and so one had to adopt the much higher σ of the core, thus predicting a very long τ_μ exceeding the Hubble time, a result confirmed [16] in 1971 when different conductivities in the NS interior were taken into account. When a larger number of pulsars became available for statistics, many authors compared the observations with population synthesis Monte Carlo models. At first [17] data seemed to support evidence for field decay, but subsequent investigations [18] [19] [20] [21] have indicated that the observed PSR population is compatible with no decay or with decay on time-scales $\tau_\mu > 10^8$ yr longer than the pulsar phase, beyond which the NSs become practically invisible. There is a way to probe indirectly the possible decay of the field by searching for those old NSs moving with low speed (< 40 km s^{-1}) that can gravitationally capture the interstellar medium and shine as dim accreting sources. As discussed in §6, their presence in the Galaxy depends on the evolution of the field.

As far as the old MSPs are concerned, these sources possess very low values of $\mu \sim 7 \times 10^{25} - 10^{27}$ G cm^3 [22] at the moment they appear as pulsars. The observations are again consistent with no field decay over their radio active phase, a result inferred from the old age of the dwarf companion stars, that implies a τ_μ at least comparable to their true galactic age [23][24][25][26]. This does not preclude from the possibility that field decay has occurred during their previous evolution: this is discussed and described in §3.4.

In the next sections we survey the models for the decay of the magnetic field both for isolated NSs and for the NSs in binaries.

3.2 Field evolution in isolated objects: spontaneous decay?

Models for magnetic field decay in isolated NSs flourished over the last decade. A first improvement was to abandon the hypothesis of a uniform conductivity in the stellar crust and core; σ increases when moving from the outer liquid layers to the deeper crystallized crust [27]. Sang & Chanmugam [28] noticed that the decay of a field residing only in the crust is not strictly exponential because of its inward diffusion toward regions of higher σ. Moreover, the decay rate depends on the (highly uncertain) depth penetrated by the initial field: the decay is more rapid if, at birth, the field resides in the outer lower density regions of the crust both because σ is lower and because τ_μ depends on the length scale of field gradients (less than a tenth of the stellar radius R if μ is confined in the crust).

The inclusion, in the calculation, of the cooling history of the NS was a second major improvement [29] [30] [31]. It confirmed the non exponential behavior of the decay, showing in addition that a slower cooling would accelerate the decay (as a warmer star has a lower conductivity). A first short phase ($\lesssim 10^6$ yr) of comparatively rapid decay was found, during which μ is reduced from values $\sim 6 \div 30 \times 10^{30}$ G cm^3, typical of young pulsars in supernova remnants, to more canonical values $\sim 1 \div 3 \times 10^{30}$ G cm^3 typical of normal PSRs. (During this phase the electron-phonon interaction dominates σ). A second phase of no decay was found lasting $10 \div 300$ Myr, followed later by a power-law decay (dominated by the interaction of the electron upon the impurities) and eventually an exponential decay.

If the field penetrates the whole star, the core ohmic decay time-scale would be too long to allow decay [32]. Thus, an alternative scenario was proposed based on the hypothesis that NSs have superfluid and superconducting cores. The angular momentum of the core of a NS is believed to be carried by [33] $N_{\rm vortex} \simeq 2 \times 10^{16}/P_{\rm sec}$ neutron vortex lines, parallel to the spin axis; the magnetic flux should be confined in $N_{\rm fluxoid}$ quantized proton flux tubes, where $N_{\rm fluxoid} \simeq 10^{32}\ B_{C,13}$, with $B_{C,13}$ being the average field strength in the core, expressed in units of 10^{13} Gauss. In 1990, Srinivasan *et al.* [34] suggested that the inter-pinning [35] between these quantized entities causes the fluxoids to be carried and deposited in the crust as the NS spins down. The magnetic flux penetrated into the crust will then decay due to ohmic dissipation and the surface magnetic field of the NS keeps decreasing until it relaxes at the value of the residual field in the core. In this framework the *spin history* of the star drives and controls the changes of the magnetic field. Assuming the same radial velocity for both the fluxoids and the vortices this model implies a time-evolution

$$\mu(t) = \mu_0\ (\ 1\ +\ t/\tau_{\rm sdd})^{-1/4} \tag{13}$$

occurring on a spin down time

$$\tau_{\rm sdd} \sim 8 \times 10^6\ \ P_0^2\ I_{45}\mu_{0,30}^{-2}\ \ {\rm yr}, \tag{14}$$

where index 0 denotes values at the onset of phase $\mathcal{E}$, and I_{45} the NS moment of inertia in units of 10^{45} g cm^2. The equation (13) models the simplest version of the *spin down driven* (sdd) decay of μ in isolated NSs. It can be refined, accounting for the non instantaneous relaxation of the surface magnetic field to the value in the core [36].

Disappointedly, until now it is not possible to perform a reliable statistical test of all the models. They predict clearly distinct evolutionary pathways only during the early stage of the pulsar life, namely during the first 10^6 yr after birth. Therefore a large sample of young NSs is requested for a comparison. Up to now, the pulsar catalog lists only an handful of such objects, as the observed PSR population is dominated by elder sources, with characteristic time of few 10^7 yr. In such situation, there is no statistical clue for rejecting the hypothesis of a non decaying field. Some of the surveys in progress, as the one running at Parkes [37], dedicated to the search of young PSRs, could ultimately solve this problem.

3.3 Magnetic field evolution in binaries: accretion driven scenario

None of the models for spontaneous decay, proposed so far for the isolated objects, can explain magnetic moments of $\sim 10^{26}$ G cm^3, which are typical of the millisecond PSRs. In order to attain these values of μ over a Hubble time, the crustal models would require currents located only in a narrow layer below a density $\rho < 6 \times 10^{10}$ gcm^{-3}, a very unlikely possibility. On the other hand, the "sdd" models would predict only a modest decrease of μ, at most of two orders of magnitude with respect to the initial values. However, one common feature groups the low-μ objects, i.e., that *all the PSRs with low magnetic moment spent a part of their life in a multiple stellar system.* Guided by this fact, many authors have tried to relate the low values of μ with the interaction between the NS and its companion star(s). Two different scenarios have been proposed so far: (i) The *accretion driven scenario* developed under the hypothesis of a *crustal magnetic field*; (ii) The *spin driven scenario* lying on the hypothesis of a *core magnetic field.*

In this section we consider scenario (i) taking the hypothesis that the currents generating the field are located deep in the crust [2]. A first observational suggestion about the effects of the accretion on a (crustal) magnetic field was given in 1986 by Taam & van den Heuvel [40]: they noticed an approximate dependence of the surface magnetic field B_s on the amount of mass accreted onto the neutron star. Shibazaki *et al.* [41] later presented an empirical formula for the decay in presence of accretion:

$$\mu \ = \ \frac{\mu_0}{1 \ + \ \Delta M_{\rm acc}/m_*} \tag{15}$$

where μ_0 represents the magnetic moment at the onset of accretion, $\Delta M_{\rm acc}$ is the amount of accreted mass and m_* a parameter to be fitted with the observations. These authors claimed that $m_* \sim 10^{-4}{\rm M}_\odot$ could reproduce the Taam & van den Heuvel's correlation. Zhang, Wu & Yang [42] [43] gave physical foundation to (15) assuming that the compressed accreted matter has ferromagnetic permeability. However, using a larger database Wijers [44] showed that these models are not fully consistent with the available data both on X-ray binaries and recycled pulsars. Romani [45] first introduced the accretion rate $\dot M$ as a second parameter in driving the magnetic moment decay in addition to $\Delta M_{\rm acc}$. He pointed out that the accretion produces two major effects: (i) heating of the crust (depending on $\dot M$), which determines a reduction of the conductivity (and in turn a hastening of the ohmic decay) and (ii) advection of the field screened by the diamagnetic accreted material. As a result, the final value of μ depends both on $\Delta M_{\rm acc}$ and on $\dot M$. Moreover, the advection of the field lines stops when $\mu \lesssim 10^{27}$ G cm^3,

[2] The onset of a thermo-magnetic instability, which transforms heat into magnetic energy at the moment of NS formation, is an effective mechanism to produce strong fields in the crust of a NS [38] [39]. Although this instability is not yet completely explored for poloidal fields, it is a plausible mechanism for the origin of a crust field which does not depend on special assumptions.

resulting in an asymptotic value for the magnetic moment (in agreement with the observations).

The existence of a bottom field (with a predicted scaling $\mu \propto \dot{M}^{1/2}$) is also possible if the currents sustaining the surface magnetic field of the NS are neutralized by the currents developed in the diamagnetic blobs of accreted matter [46]. In this case the decay of μ typically ends when the accretion disc skims the NS surface [47].

The first fully consistent theoretical calculations of the accretion-induced-decay of the crustal magnetic field of a NS were performed by Urpin, Geppert & Konenkov [48] [49] [50] [51]. The basic physical mechanism is the diffusion (ohmic decay of currents) and advection of the magnetic field sustained by currents circulating in the non superconducting crust. The magnetic field evolution is calculated according to the induction equation:

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{c^2}{4\pi}\nabla \wedge \left(\frac{1}{\sigma}\nabla \wedge \boldsymbol{B}\right) + \nabla \wedge (\boldsymbol{v} \wedge \boldsymbol{B}) \tag{16}$$

where $\boldsymbol{B}$ is the magnetic induction and $\boldsymbol{v}$ is the velocity of the moving fluid ($=0$ in a non accreting NS, $|\boldsymbol{v}| = \dot{M}/4\pi r^2\rho$ for a radial approximation of the accreted fluid flow). In their calculation, the superconducting NS core is assumed to expel the magnetic field (in the following we will refer to this boundary condition for the field at the crust-core interface as BCI); the induction equation is solved for a dipolar field and σ is given as in [52].

Accretion affects the ohmic decay in two ways: (i) *heating the crust,* so reducing σ; (ii) *transporting matter and currents* toward the core-crust boundary. The relative importance of these two effects depends on other physical parameters, such as the initial depth penetrated by the currents, the impurity content (Q), the accretion rate, the duration of the accretion phase and the equation of state of the nuclear matter. Coupling the magnetic history of the star to its spin evolution, Urpin, Geppert & Konenkov [53] found that *neutron stars born with standard magnetic moments and spin periods can evolve to low-field rapidly rotating objects*, as illustrated in Figure 1.

The behavior of currents and the response of nuclear matter to an interior field, at the interface between the crust and the core of a NS (typical density $\simeq 2 \times 10^{14}$ g cm^3), is still an open issue for the theorists. In view of these uncertainties, Konar & Bhattacharya [54] chose a different boundary condition at the crust-core interface for solving the equation (16). They noticed that the deposition of accreted matter on top of the crust can imply the assimilation of original current-carrying crustal layers into the superconducting core of the NS. In particular they assumed [54] the *newly formed superconducting material to retain the magnetic flux* in the form of Abrikosov fluxoids [15] rather than to expel it through the Meissner effect (in the following this boundary condition will be labeled as BCII). Within this model, accretion produces a third effect on the evolution of μ: (iii) the *assimilation of material into the core*, where the conductivity is huge, freezes the decay.

As illustrated in Figure 2, also *this hypothesis leads to a reduction of the surface magnetic field of more than 4 orders of magnitude* in 10^8 yr, explaining

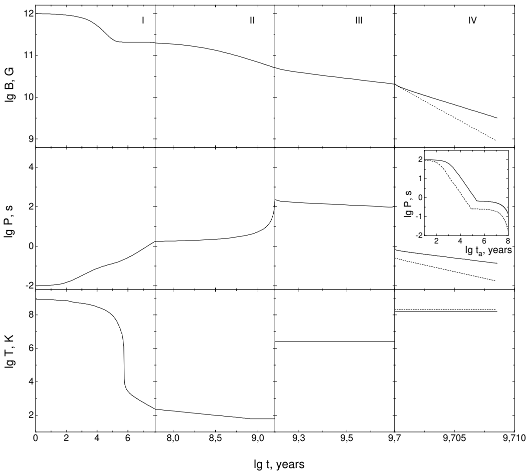

Fig. 1. Magnetic, rotational and thermal evolution vs time of a 1.4 $M_\odot$ NS with $\mu = 10^{30}$ G cm^3 at birth, as calculated in [53], under BCI. The adopted equation of state is PS (a stiff one). The initial depth of the currents is 10^{13} g cm^{-3} and the impurity parameter $Q = 0.01$. Phase I and II refers to $\mathcal{E}$ and $\mathcal{P}$ respectively. The value of $\dot{M}$ from the stellar wind (referred to as phase III of $\mathcal{A}$) is $2/3 \times 10^{-8} \dot{\mathrm{M}}_{\mathrm{E}}$, and the lifetime of the companion star on the main sequence is 5×10^9 yr. During the disc accretion phase (as referred to IV), $\dot{M}$ is $2/3 \times 10^{-3} \dot{\mathrm{M}}_{\mathrm{E}}$ (*solid line*) or $2/3 \times 10^{-2} \dot{\mathrm{M}}_{\mathrm{E}}$ (*dotted line*). Phase IV lasts 10^8 yr. The inserted panel enlarges the spin evolution during the very short phase at the boundary between phase III and IV, that is when disc accretion sets in.

the low μ and its possible freezing in the millisecond pulsar population. Likewise for BC I, it emerges that the decay of μ depends not only on the total accreted mass, but on the accretion rate itself. However, in this case the higher the accretion rate is, the stronger is the pull of crustal material into the core resulting in an earlier freezing of the decay. In a subsequent paper [55], it was claimed that such a positive correlation between $\dot{M}$ and the final value of μ is supported by some observational evidence [56] [57].

Other refinements of the accretion-induced decay scenario include the effects of a non spherical symmetric accretion [58], the evolution of multipoles at the NS surface [58], the post-accretion increase of μ due to the re-diffusion of the buried field towards the surface [59], and the relativistic corrections to the decaying history [60]. All together, they produce only slight changes to the aforementioned results.

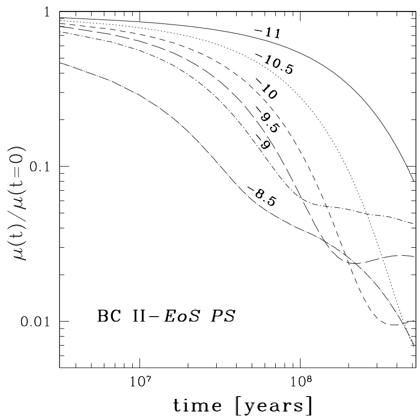

Fig. 2. Evolution, under BCII, of the surface magnetic field of a 1.4 $M_\odot$ NS undergoing accretion at six different levels. From top to bottom $\dot{M}_{acc} = 2/3$ of $10^{-5}; 10^{-4}; 10^{-3}; 10^{-2} \dot{M}_E$, and $1.3 \times 10^{-2}; 2/3 \times 10^{-1}$ $\dot{M}_E$. The flattening of the curves for the higher $\dot{M}$ is clearly visible [54].

3.4 Magnetic field evolution in binaries: spin driven scenario

Both the simple idea of the flux conservation during the gravitational collapse and the action of a dynamo in the convective proto-neutron star [61] lead to the existence of a magnetic field penetrating the whole NS. Due to the huge conductivity of the core matter, no ohmic decay occurs during a Hubble time and so other effects have been invoked to account for the low field NSs.

Besides magneto-dipole braking, a NS experiences a longer and more significant phase of spin-down, phase $\mathcal{P}$ (described in §2) which enhances the effect of the spin-down driven scenario sketched in §3.2. This is particularly important when the NS lives in a binary system. Miri & Bhattacharya [36] and Miri [62] explored the case of low mass systems showing that at the end of $\mathcal{P}$ and $\mathcal{A}$ phases the magnetic moment μ has decayed, relative to its initial value μ_0, by a factor scaling with $P_0/P_{\max}$ where P_0 is the initial rotational period and $P_{\max}$ the longest period attained during the phase where $\mathcal{P}$ is active ($\mathcal{P}$ results from the interaction of the companion wind with the NS). Assuming that the star spins-down to $P \lesssim 10^3$ sec, a residual magnetic moment is

$$\mu_{\rm final} \simeq \frac{0.1 \div 1 \text{ sec}}{1000 \text{ sec}} \mu_0 \simeq 10^{-4} \div 10^{-3} \mu_0 \simeq 10^8 \div 10^9 \text{ G cm}^3 \qquad (17)$$

compatible with the values observed for MSP population.

Recently, Konar & Bhattacharya [63] re-examined this scenario in the case of accreting NSs, incorporating the microphysics of the crust and the material movements due to the accretion. They concluded that the model can reproduce the values of μ observed in the low-mass systems only for large values of the unknown impurity parameter $Q \gtrsim 0.05$ (in contrast with the *accretion driven decay* models demanding for a $Q \lesssim 0.01$). If the wind accretion phase is short or absent, Q should even exceed unity. A more serious objection to this model was risen by Konenkov & Geppert [64] who pointed out that the motion of the proton flux tubes in the core leads to a distortion of the field structure near the crust-core interface and this in turn creates a back-reaction of the crust on the fluxoids expulsion: it results that (i) the sdd can be adequate for describing the μ-evolution only for weak initial magnetic moments ($\mu_0 \lesssim 10^{29}$ G cm^3) and (ii) the predicted correlation $\mu_{\rm final} \propto P_{\rm max}^{-1}$ is no more justified.

Finally, we have to mention another class of models in which the magnetic evolution is strictly related with the spin history of the star: in 1991 Ruderman [65] [66] proposed that the coupling among P and μ could take place via crustal plate tectonics. The rotational torques acting on the neutron star cause the crustal plates to migrate, thus dragging the magnetic poles anchored in them. As a consequence the effective magnetic dipole moment can strongly vary, either in intensity (decreasing or growing) and in direction, suggesting a tendency to produce an overabundance of both orthogonal (spin axis $\perp$ magnetic axis) and nearly aligned rotators (spin axis $\|$ magnetic axis). Current observation of disc population MSPs seem compatible with this feature [67]. Moreover the crust-cracking events could account for the glitches seen in young PSRs [68]. A more extended description of the variety of the spin driven decay models can be found for example in [69].

4 Evolutionary pathways in various environment

As illustrated in the previous sections, the magnetic field and spin evolution in phases $\mathcal{E}$, $\mathcal{P}$ or $\mathcal{A}$ is a sensitive function of the environment. For the isolated NSs, the intrinsic velocity distribution plays, in addition, an important role. Thus, we here describe the NS evolution in the two main environments: in low mass binaries and in the ISM medium.

4.1 Binaries

The NSs in binaries experience a complex evolution which is tightly coupled to the orbital and internal evolution of the companion star [70]. The powerful pulsar wind initially sweeps the stellar wind away and the NS spends its lifetime in the $\mathcal{E}$ phase. With the weakening of the electro-magnetic pressure with increasing period, an extended $\mathcal{P}$ phase establishes (lasting $10^8 - 10^9$ yr) during which the NS is spun down further to periods of $\sim 10^2 - 10^4$ seconds in its interaction with the stellar wind of the companion star. When the period $P_{\mathcal{P}\to\mathcal{A}}$ is attained, accretion sets in down to the NS surface and we could possibly trace this phase identifying the X-ray emission from the wind fed NS. Here the NS can accrete with no exchange of angular momentum with the incoming matter unless the magnetic field decays due to either accretion induced or spin induced decay. The NS can slide along the corresponding equilibrium spin up line. As soon as the donor star evolve into a giant state, it can fill its Roche lobe (and matter overflows from the inner Lagrangian point forming an accretion disc. When a disc establishes (on the viscous time scale) large accretion rates are available and from this moment on recycling starts. In the Roche lobe overflow phase (RLO) the NS can be re-accelerated to millisecond periods while the field decays down to values of $10^8 - 10^9$ G cm^3. The mass transfer and orbital evolution are complex to model and it is now believed that a significant fraction of the mass lost by the donor does not accrete onto the NS, but is ejected from the system [71].

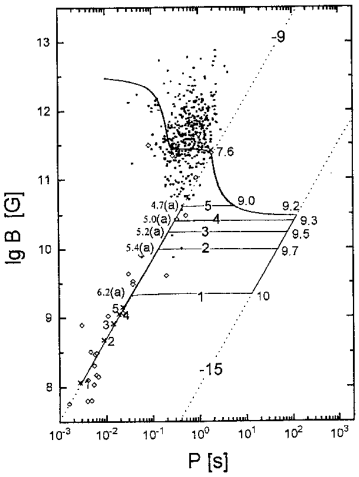

Fig. 3. The evolutionary tracks of NSs in the plane $B - P$ for different durations of the evolution (labeled by the numbers on the right of the curves, in $\log_{10}$ yr) during phases I($\mathcal{E}$) + II($\mathcal{P}$)+ III ($\mathcal{A}$) + IV($\mathcal{A}$). The initial surface magnetic field is 3×10^{12} G, $Q = 0.03$, and the other parameters as in Figure 1 [51].

Once reached the spin-up line at the current accretion rate, the NS loads matter acquiring angular momentum at the rate of field decay. Recycling ceases when the donor star has evolved beyond the red giant phase and a dwarf remnant is left over stellar evolution, or when the binary as become detached, so that the donor underfills its Rocche lobe [70]. If accretion ends abruptly, the NS may avoid phase $\mathcal{P}$, transiting directly to $\mathcal{E}$ and possibly re-appearing as an active radio pulsar. In low mass binaries where the donor star is a low main sequence star, evolution proceeds in a rather ordered way: from $\mathcal{E}$ to $\mathcal{P}$ (from the stellar wind), to $\mathcal{A}$ (from wind fed accretion), to $\mathcal{A}$ again (from a Keplerian disc; RLO), as shown in Figure 3. Eventually the NS tranits to $\mathcal{E}$ or $\mathcal{P}/\mathcal{E}$ when accretion halts. Observational and theoretical considerations hint for a large decay of the magnetic moment μ in phase $\mathcal{A}$, particularly during RLO, as outlined also in

Figures 1 and 3. The recycling scenario will be applied in §5 to address the problem of the existence of sub-millisecond pulsars.

4.2 Isolated neutron stars

At birth, isolated NSs experience phase $\mathcal{E}$. Further evolution depends on the NS velocity relative to the ISM, the value of the ISM density and the magnetic field intensity. The large mean kick velocities acquired at birth ($\langle V\rangle \sim 300$ km s^{-1}, [20] [21][72][73]) can reduce the extent of magnetic torques in phase $\mathcal{P}$ and can impede accretion fully (as $\dot{M} \propto v_{\rm rel}^{-3}$ and here $v_{\rm rel}$ coincides with $\langle V\rangle$). It is thus possible that phase $\mathcal{E}$ never ends. This occurs when the NS speed V is as high as $V > 100\,\mu_{30}\,n^{1/2}$ km s^{-1} : This speed is derived estimating the duration of phase $\mathcal{E}$ at $P_{\mathcal{E}\to\mathcal{P}}$ (eqs. 2 and 5)

$$\tau_{\mathcal{E}\to\mathcal{P}} = \tau_{\mathcal{E}}(P_{\mathcal{E}\to\mathcal{P}}) \sim 10^9 n^{-1/2} V_{10} \mu_{30}^{-1} \mathrm{yr} \qquad (18)$$

imposing $\tau_{\mathcal{E}\to\mathcal{P}} \sim 10^{10}$ yr (in eq. 18, V_{10} and μ_{30} are the velocity and the magnetic moment in units of 10 km s^{-1} and 10^{30} G cm^3, respectively; hereon we will denote the normalizations as subscripts, for simplicity). Clearly, only the slowest NSs can loop into the accretion phase under typical ISM conditions (where $n \lesssim 1$ cm^{-3}). (Molecular clouds seem a privileged site for phase $\mathcal{A}$ [74], but the probability of crossing these high density regions is relatively low and shortlived.)

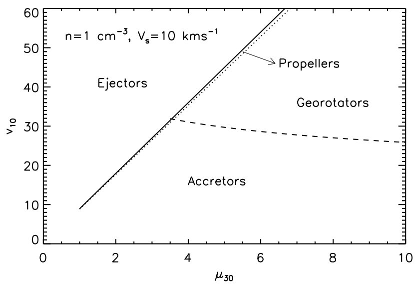

Fig. 4. Phases $\mathcal{E}, \mathcal{P}, \mathcal{A}$ and $\mathcal{G}$, in the $V_{10} - \mu 30$ plane for isolated NSs moving in a ISM of density $n = 1$ cm^{-3} and a sound speed of 10 km s^{-1}, from [101].

Typically, in the ISM

$$P_{\mathcal{E}\to\mathcal{P}} \sim 10\,n^{-1/4} V_{10}^{1/2} \mu_{30}^{-1}\,\mathrm{s} \quad \text{and} \quad P_{\mathcal{P}\to\mathcal{A}} \sim 2.5\times 10^3 n^{-3/4} V_{10}^{1/7} \mu_{30}^{6/7}\,\mathrm{s}, \qquad (19)$$

if the magnetic field is constant in time. As indicated in equations (18) and (19), the NS evolution is guided by the two key parameters, V and μ (at fixed ISM's density). This is quantitatively illustrated in Figure 4 where the NS's phases are identified in the V vs B plane. A strong constant magnetic field ($\mu \sim 10^{30}$ G cm^3) implies large magneto-dipole losses that decelerate rapidly the NS to favor its entrance in phase $\mathcal{A}$ (even when $V \sim 300$ km s^{-1}), despite the long period of the $\mathcal{P} \rightarrow \mathcal{A}$ transition (note eq. 19).

What are the consequences of field decay on the evolution? Whether field decay enhances or reduces the probability of a transition beyond $\mathcal{E}$ is a subtle question that has recently been partly addressed [75][76][77][78]. Two competing effects come into play when B decays: (i) the spin-down rate slows down because of the weakening of B, causing the NS to persist longer in state $\mathcal{E}$; (ii) the periods $P_{\mathcal{E}\rightarrow\mathcal{P}}$ and $P_{\mathcal{P}\rightarrow\mathcal{A}}$ instead decrease with B, and this acts in the opposite sense.

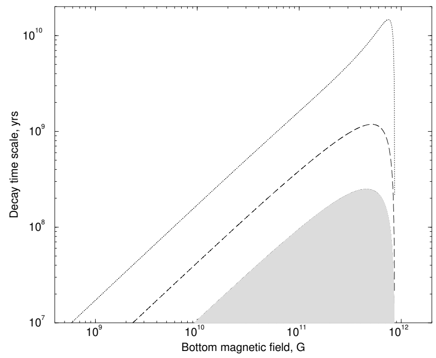

Fig. 5. Loci, in the $\tau_\mu - B_{\rm bo}$ plane, where phase $\mathcal{E}$ lasts longer than 10^{10} yr. The curves refer (from top to bottom) to initial fields of 5×10^{11} (dotted line), 10^{12}(dashed line), 2×10^{12} G (hatched region), for $v_{\rm rel} + c_s = 40$ km s^{-1}, and ISM density $n = 1$ cm^{-3}.

To explore this delicate interplay, Popov & Prokhorov [78] studied a toy model where a field is exponentially decaying on a scale τ_μ from its initial value μ_0 down to a bottom value $\mu_{\rm bo}$. Under this hypothesis, the Ejector time scale (with a decaying μ) can be estimated analytically to give

$$\tau_{\mathcal{E},\mu} = \begin{cases} -\tau_\mu \ln \left[\dfrac{\tau_\mathcal{E}}{\tau_\mu} \left(1 + \dfrac{\tau_\mu^2}{\tau_\mathcal{E}^2} \right)^{1/2} - 1 \right] & \tau_{\mathcal{E},\mu} < \tau_{\rm bo} \\ \tau_{\rm bo} + \dfrac{\mu_0}{\mu_{\rm bo}} \tau_\mathcal{E} - \dfrac{1}{2} \tau_\mu \dfrac{\mu_0}{\mu_{\rm bo}} \left(1 - e^{2\tau_{\rm bo}/\tau_\mu} \right) & \tau_{\mathcal{E},\mu} > \tau_{\rm bo} \ , \end{cases} \qquad (20)$$

where $\tau_{\rm bo} = \tau_d \ln(\mu_0/\mu_{\rm bo})$ is the time when the bottom field is attained and $\tau_\mathcal{E}$ is from equation (2) for a constant field. Figure 5 shows the loci where the time spent in phase $\mathcal{E}$ equals the age of the Galaxy, i.e., 10 Gyr, as a function of the bottom field $B_{\rm bo}$ and of the decay time-scale τ_μ. The “forbidden” region lies just below each curve. As it appears from the Figure, the interval over which the NS never leaves stage $\mathcal{E}$ is non negligible: It widens when μ_0 is decreased, due to the weakness of the magneto-dipole torque. Large as well as very low bottom fields do not constrain τ_μ and permit entrance to phase $\mathcal{A}$. When $\mu_{\rm bo}$ is very low, there is a turn-off of all magnetospheric effects on the inflowing matter and matter accretes promptly.

Similarly, Colpi *et al.* [75] considered a model in which the spin evolution causes the core field to migrate to the crust where dissipation processes drive ohmic field decay (Sect. 3). In this circumstance, the entrance to phase $\mathcal{A}$ is less likely if $\tau_\mu \sim 10^8$ yr, but possible otherwise. In summary, field decay can hinder the stars in phase $\mathcal{E}$ if its decay is somewhat "fine-tuned", while a fast decay would drive them into $\mathcal{A}$ promptly. The strongest theoretical argument against phase $\mathcal{A}$ remains however the high kick velocity that the NSs acquire at birth (see the review of Lai in this book). There remain nevertheless open the possibility that weak field accreting NSs exist in the Sun's vicinity.

5 A population synthesis approach to the formation of *sub*-MSPs

This section is devoted to the statistical study of the old NSs in binaries in the aim at exploring the recycling process and the possibility that ultra fastly spinning NSs can form in these systems. A similar approach is presented in §6 for the isolated NSs in the Galaxy.

PSR1937+21, at present, is the NS having the shortest rotational period $P_{\rm min} = 1.558$ ms ever detected. Despite its apparent smallness, $P_{\rm min}$ is not a critical period for NS rotation: as shown by Cook, Shapiro & Teukolsky [79], the period $P_{\rm min}$ is longer than the limiting period, $P_{\rm sh}$, below which the star becomes unstable to mass shedding at its equator, irrespective to the adopted equation of state (EoS) for the nuclear matter. Table 1 just shows the values of $P_{\rm sh}$ for a set of EoS and the corresponding values of accreted baryonic mass. It shows how sensitive is $P_{\rm sh}$ to the EoS, indicating that its determination is important for our understanding of nuclear processes in dense matter.

The other important processes which intervene in accelerating a NS up to $P_{\rm sh}$ are the evolution of the mass transfer rate from the low mass companion star and the evolution of the NS magnetic moment μ [80]. Within the recycling scenario [81][82] all schemes suggested for the origin and the evolution of μ allow the accreting NSs to reduce their μ and P to the values that are characteristic of MSPs. However, due to the large volume of the parameters in the mass transfer scenario and to the strong coupling between it and the field evolution of the NS in a low mass binary (LMB), only through a statistical approach it is possible to establish how efficient the recycling process is in spinning a NS to $P < P_{\rm min}$. This was recognized first by Possenti *et al.* [83][84], who carried on statistical analyses of NSs in the millisecond and sub-millisecond range, using a Monte Carlo population synthesis code with ~ 3000 particles. It accounts for the evolution during the early phases when the NS in the LMB behaves as if isolated ($\mathcal{E}$ and $\mathcal{P}$) and later when fed by wind accretion ($\mathcal{A}$). The mass transfer

Table 1. Disc accretion onto a 1.4 $M_\odot$ neutron star

EoS	Fate	T/W_{fin}	ΔM_B	$M_{G,fin}$	P_{fin}	$M^{static}_{G,max}$	$M^{rotating}_{G,max}$	P^{abs}_{min}
F	Collapse	0.034	0.172	1.52	0.72	1.46	1.67	0.47
A	MassShed	0.120	0.428	1.77	0.60	1.66	1.95	0.47
E	MassShed	0.115	0.414	1.76	0.66	1.75	2.05	0.48
AU	MassShed	0.126	0.446	1.79	0.70	2.13	2.55	0.47
D	MassShed	0.111	0.405	1.76	0.73	1.65	1.95	0.57
FPS	MassShed	0.114	0.416	1.76	0.75	1.80	2.12	0.53
UT	MassShed	0.119	0.429	1.78	0.75	1.84	2.19	0.54
UU	MassShed	0.121	0.436	1.78	0.78	2.20	2.61	0.50
C	MassShed	0.103	0.389	1.74	0.89	1.86	2.17	0.59
N*	MassShed	0.130	0.484	1.84	1.08	2.64	3.22	0.68
L	MassShed	0.116	0.443	1.80	1.25	2.70	3.27	0.76
M	MassShed	0.091	0.367	1.74	1.49	1.80	2.10	0.81

The second and the third columns contain the fate of the NS at the end point of recycling and the resulting ratio of kinetic energy over gravitational energy. The fourth and fifth columns report the total accreted baryonic mass and the final gravitational mass (in units of solar masses). The sixth column lists the final attained rotational periods. The seventh column collects the values of the maximum mass for a non-rotating spherical configuration, the eighth for a maximally rotating star and its corresponding minimum period for stability (ninth column). The table is from Cook, Shapiro & Teukolsky [79].

during RLO (again $\mathcal{A}$) is modelled considering a range of accretion rates which is close to the one observed. The population synthesis code follows also the last radio-pulsar phase, whose duration is chosen from a flat probability distribution in the logarithm of time (see Table 2 for a summary of the parameters used in the population synthesis code). The model incorporates the detailed physics of the evolution of a crustal magnetic field (as discussed in §3.3, using BCI and BCII to mimic expulsion or assimilation of the field in the NS core), and includes the relativistic corrections [80] necessary to describe the spin-up process.

Evolution is followed also beyond the RLO $\mathcal{A}$ phase when accretion terminates. The increasing evidence that NSs in LMBs may suffer phases of transient accretion (perhaps due to thermal-viscous instabilities in an irradiation dominated disc [85][86]) is suggestive that mass transfer onto a NS may not stop suddenly: the star probably undergoes a progressive reduction of the mean accretion rate, modulated by phases of higher and lower accretion. This in turn can start a cycle of $\mathcal{P}$ phases which in principle could vanish the effect of the previous spin-up. With the aim of exploring the effect of a decaying $\dot{M}$ on the population of fastly spinning objects, two possibilities have been investigated: a

Table 2. Population syntheses parameters

Physical quantity	Distribution	Values	Units
NS period at $t_0^{\rm RLO}$ (*)	Flat	$1 \rightarrow 100$	sec
NS μ at $t_0^{\rm RLO}$ (*)	Gaussian	Log$< \mu_0 >$=28.50 ; σ=0.32	G cm^3
$\dot{m}$ in RLO phase ($^\sharp$)	Gaussian	Log$< \dot{m} >$= 1.00 ; σ=0.50	$\dot{\rm M}_{\rm E}$
Minimum accreted mass	One-value	0.01	$\rm M_\odot$
RLO accretion phase time (†)	Flat in Log	$10^6 \rightarrow \tau_{RLO}^{max}$(‡)	year
MSP phase time	Flat in Log	$10^8 \rightarrow 3 \times 10^9$	year

(*) $t_0^{\rm RLO}$ = initial time of the Roche Lobe Overflow phase
($\sharp$) baryonic accretion rate during the Roche Lobe Overflow phase
($\dagger$) a Maximum accreted Mass of $0.5 M_\odot$ is permitted during the RLO phase
($\ddagger$) max duration of the RLO phase; explored values: $5{\times}10^7$ yr - 10^8 yr - $5{\times}10^8$ yr

persistent accretion for a time τ_{RLO} or a persistent accretion for a shorter time followed by a transient phase mimicking the quenching of the mass transfer. The quenching of accretion has been modelled as a power law decay for $\dot{M}$ with index Γ varying from $1 \rightarrow 10$ (the last value is representative of an almost sudden switch off).

Figure 6 collects the fractional distribution of the recycled model NSs using the set of parameters reported in Table 2 with $\tau_{RLO}^{\rm max} = 5 \times 10^8$ yr. Guided by the values of $P_{\rm min}$ and $\mu_{\rm min}$ (the shortest rotational period observed in PSR 1937+21 and the weakest magnetic moment observed in PSRJ2317+1439), the particles are divided in four groups. Those filling the first quadrant ($P \geq P_{\rm min}$ and $\mu \geq \mu_{\rm min}$) behave as the known MSPs. Also the objects belonging to the second quadrant ($P < P_{\rm min}$ and $\mu \geq \mu_{\rm min}$) should shine as PSRs [87]. The effective observability of the objects in the third quadrant ($P < P_{\rm min}$ and $\mu < \mu_{\rm min}$) as radio sources represents instead a challenge for the modern pulsar surveys. Most of them will be above the “death-line” [88], and might have a bolometric luminosity comparable to that of the known MSPs. Thereafter we shortly refer to *sub*-MSPs as to all objects having $P < P_{\rm min}$ and μ above the “death-line”. Objects in the fourth quadrant ($P \geq P_{\rm min}$ and $\mu < \mu_{\rm min}$) are probably radio quiet NSs, because they tend to be closer to the theoretical “death-line”, and they are in a period range which was already searched with good sensitivity by the radio surveys.

The four crosses are for two representative EoSs (a very stiff EoS with a break-up period 1.4 ms and a mildly soft one, with $P_{\rm sh} \simeq 0.7$ ms) and for two different boundary conditions (BCI & BCII) for the magnetic field at the crust-core interface. It appears that objects with periods $P < P_{\rm min}$ are present in a statistically significant number. In effect, *a tail of potential sub-*MSPs *always emerges* for any reliable choice of the parameters listed in Table 2.

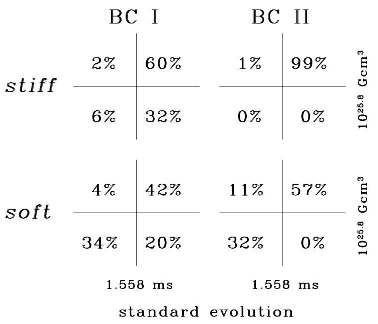

Fig. 6. Distributions of the synthesized NSs, derived normalizing the sample to the total number of model stars with $P < 10$ ms and arbitrary value of μ. The $\mu - P$ plane is divided in four regions. As a guideline the upper left number in each cross gives the percentage of objects having $P < P_{\rm min}$ and $\mu > \mu_{\rm min}$ (the typical variance is about 1%).

In the synthetic sample for the *mild-soft* EoS, the "barrier" at $P_{\rm sh} \simeq 0.7$ ms is clearly visible in Figure 7 (solid lines): the *mild-soft EoS gives rise to period-distributions that increase rather steeply toward values smaller than 2* ms, irrespective to the adopted BCs. Instead, the boundary condition affects the distribution on μ: BCII produces a smaller number of objects with low field, as the field initially decays but, when the currents are advected toward the crust-core boundary, their decay is halted and the field reaches a bottom value.

Even the *very stiff* EoS permits periods $P < P_{\rm min}$, but the "barrier" of mass shedding (at $P_{\rm sh} \simeq 1.4$ ms) is so close to $P_{\rm min} = 1.558$ ms that only few NSs reach these extreme rotational rates. Moreover the period distribution for the stiff EoS is much flatter than that for the soft-EoS, displaying a broad maximum at $P \sim 3$ ms. It was recently claimed that X-ray sources in LMBs show rotational periods clustering in the interval $2 \rightarrow 4$ ms [89] [90]. This effect could be explained introducing a fine tuned relation between μ and $\dot{M}$ ($\mu \propto \dot{M}^{1/2}$ [89]). Alternatively, gravitational waves emission has been invoked [91][92]. Here we notice that such a clustering can be a natural statistical outcome of the recycling process if the EoS for the nuclear matter is stiff enough.

Many physical ingredients necessary to fully describe the propeller induced spin-down of a NS at the end of the RLO phase are poorly known or difficult to assess (e.g. the exact law for the decrease of the mass transfer rate or the efficiency in the extraction of the angular momentum from the NS to the propelled matter). Parametrizing the most uncertain quantities, the largest effect occurs for a propeller phase lasting $\sim 50\%$ of the RLO phase with a power-law index $\Gamma \sim 8$. Figure 7 reports the period distributions (dashed areas) when such a strong propeller phase is included. We note that *a strong propeller can*

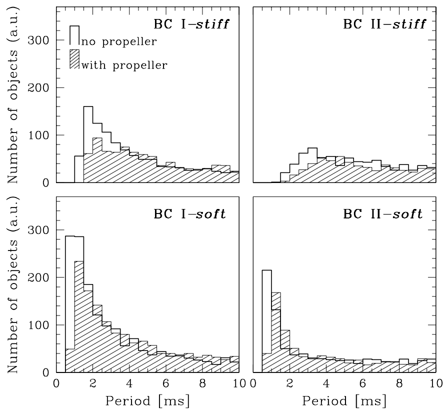

Fig. 7. Calculated distribution of millisecond NSs as a function of the spin period P (μ is let vary over the whole range). *Solid line* denotes the distribution in absence of propeller, whilst *dashed area* denotes the case of a strong propeller effect. The absolute number of objects is in arbitrary units.

threaten the formation of NSs with $P < P_{\rm min}$ *and* $\mu > \mu_{\rm min}$ in the case of the very stiff EoS, whilst *for the mild-soft EoS the distributions preserve a maximum just about* $P_{\rm min}$.

This statistical analysis provides also information on the NS mass distribution as a function of P at the end of evolution. The initial NS gravitational mass, in the evolutionary code, was set at $M_G = 1.35$ M$_\odot$ (according to the narrow Gaussian distribution with $\sigma = 0.04$ M$_\odot$ resulting from measuring the mass of the NSs in five relativistic NS-NS binary systems [93]). Figure 8 clearly shows that the observed millisecond population should have undergone a mass load of $\lesssim 0.1$M$_\odot$ during recycling. This is consistent with the few estimates of the masses of millisecond pulsars in low mass binaries [94] and raises the problem of explaining how the NS can get rid of the bulk of the mass $(0.5 - 1.5$ M$_\odot)$ released by the companion during the RLO phase [71].

Figure 8 suggests that the mass function steepens toward high values only when P falls below $\sim P_{\rm min}$, approaching $M \sim 1.7 \div 1.8$ M$_\odot$. That is a straight consequence of a results already pointed out by Burderi *et al.* [80]: a large mass deposition (at least $\gtrsim 0.25$M$_\odot$) is required to spin a NS to ultra short periods, as illustrated in Figure 9. The action of the propeller during the evolution has the effect of reducing the mass infall: the mass distribution is only slightly affected for the mild-soft EoS, while for the stiff EoS the difference is more pronounced.

In summary, the wandering among the various phases can account for the interesting phenomenology of the MSPs observed and opens the possibility that even more extreme objects like the *sub*-MSPs exist in the Galaxy.

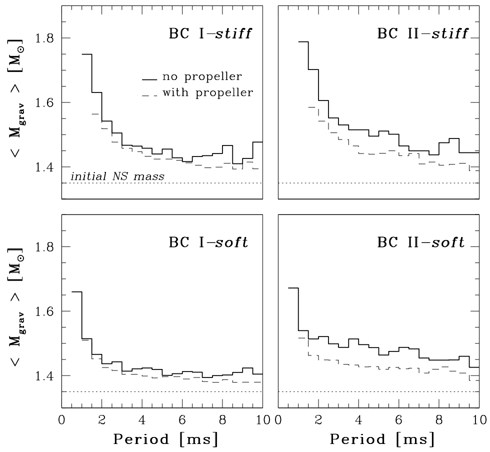

Fig. 8. Average gravitational masses of the re-accelerated NSs as a function of their final spin period P. μ varies over the whole range and the synthesized NSs are binned in 0.5 ms wide intervals. The initial mass of the static NSs is set equal to $1.35\mathrm{M}_{\odot}$ in all the cases. *Thick solid line* denotes the mass-distribution in absence of propeller, whereas *thin dashed line* with a strong propeller included.

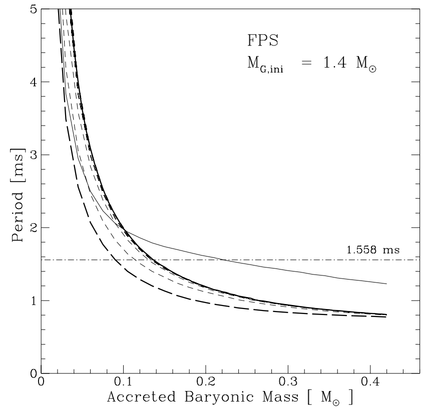

Fig. 9. Period versus Accreted Baryonic Mass for a magnetized NS using FPS-EoS and an initial gravitational mass of 1.40 $\mathrm{M}_{\odot}$. The different pathways (a sample is represented by the *dashed lines*) define a strip, which narrows towards shorter periods. The strip is upper bounded by the evolutionary path for an unmagnetized NS (*bold solid line*). The *bold long dashed line* refers to the evolution along the spin-up line and is calculated assuming a tuned torque function (which maximizes the efficiency of the spin-up process). The *thin solid line* refers to a very slow decay of μ, implying larger mass depositions in order to attain very short spin periods.

6 The NS census in the Milky Way

The isolated NSs follow a completely different evolution pattern. Generally, over the Hubble time, they cover large portion of the Milky Way and thus explore regions where the ISM is inhomogeneous looping among the various phases erratically. With a population synthesis model, Popov *et al.*[101] traced their evolution in the ISM, and in the Milky Way potential. The model NSs, born in the galactic plane at a rate proportional to the square of the ISM density, have initially short spin periods, magnetic fields clustered around 10^{12} G, and spatial (kick) velocities that can be drawn for a Maxwellian distribution (with mean velocity modulus $\langle V \rangle$ treated as a parameter).

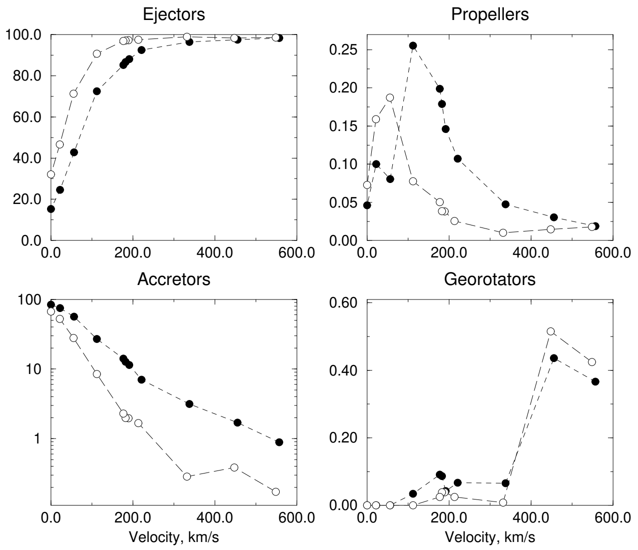

Fig. 10. Fractions of NSs (in percents) in phases $\mathcal{E}, \mathcal{PA}$ and $\mathcal{G}$ versus the mean kick velocity $\langle V \rangle$, for a constant magnetic moment $\mu = 0.5 \times 10^{30}\,\mathrm{G\,cm}^3$ (open circles) and $\mu = 10^{30}\,\mathrm{G\,cm}^3$ (filled circles); typical statistical uncertainty for $\mathcal{E}$ and $\mathcal{A} \sim$ 1-2%.

The collective properties are illustrated in Figure 10 for two initial values of $\mu_0 = 0.5 - 1 \times 10^{30}$ G cm^3: For a non evolving field most of isolated NSs spend their lives as Ejectors and there is no possibility to observe them. Propellers are shortlived [7], and Georotators are rare. A tiny fraction (a few percents) on the NSs are in the Accretor stage if $\langle V \rangle$ is above 200 km s^{-1}. Only in the unrealistic case of a low mean velocity, the bulk of the population would be in stage $\mathcal{A}$.

As illustrated in Figure 11, in phase $\mathcal{A}$, the mean value of $\dot{M}$ is a few 10^9 g s^{-1} and, among the Accretors, the typical velocity clusters around 50 km s^{-1}, and the luminosity $L \sim (GM/R)\dot{M}$ around 10^{29} erg s^{-1}. Accreting isolated NSs are "visible", but they would be extremely dim objects emitting predominantly in the soft X-rays, with a polar cap black body effective temperature of 0.6 eV. What can we learn when comparing the theoretical census with the observations? We can discover if field decays in isolated objects, if the population is devoid of slow objects, and if the stars effectively spin down due to the unavoidable interaction with the tenuous ISM. The search of accreting isolated NSs is thus compelling [102][103][104][77].

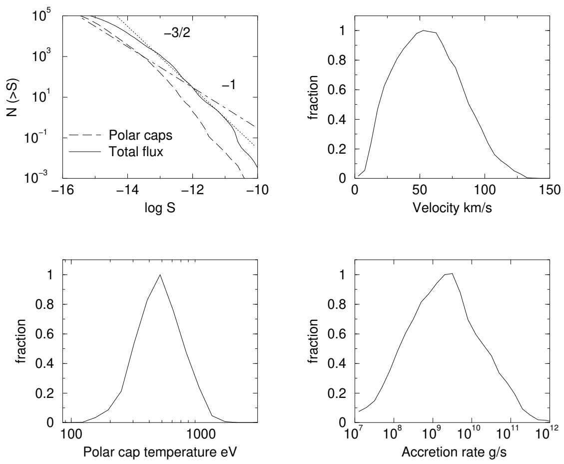

Fig. 11. Upper left panel: the log N-log S distribution for Accretors within 5 kpc from the Sun. Dashed (solid) curve refers to emission from polar cap (entire NS surface) in the range 0.5-2.4 keV: straight lines with slopes -1 and -3/2 are included for comparison. Fro top right to bottom right: the velocity V, effective polar cap temperature and accretion rate $\dot{M}$ distributions for Accretors [106]

6.1 Accreting isolated neutron stars in the *Rosat* Sky?

Despite intensive observational campaigns, no irrefutable identification of an isolated accreting NS has been presented so far. Six soft sources have been found in *Rosat* field [77], identified as isolated NSs from the optical and X-ray data. Observations, however, do not permit to unveil the origin of their emission, yet. These sources could be powered either by accretion or by the release of internal energy in relatively young ($\approx 10^6$ yr) cooling NSs [3]. Although relatively bright (up to $\approx$ 1 $\mathrm{cts\,s^{-1}}$), the proximity of the sources (inferred from the column density in the X-ray spectra) makes their luminosity ($L \approx 10^{31}$ $\mathrm{erg\,s^{-1}}$) near to that expected from either a close-by cooling NS or from an accreting NS, among the most luminous. Their X-ray spectrum is soft and thermal, again as predicted for both Accretors and Coolers.

[3] see [77] for an updated review, the description of the sources and a complete reference list.

Can a comparison with theoretical expectations help in discriminating among the two hypothesis? First, the paucity of very soft X-ray sources in the *Rosat* fields (in comparison with earlier expectation [103][104]) is indicating that Accretors are rare objects. If these six sources are indeed accreting, this implies that the average velocity of the isolated NSs population at birth has to exceed ~ 200 km s^{-1} (a figure, which is consistent with that derived from radio pulsars statistics [105]). In addition, since observable accretion–powered isolated NSs are (intrinsically) slow objects, these results exclude the possibility that the present velocity distribution of NSs is richer in low–velocity objects with respect to a Maxwellian. Thus, despite the fact that in our Galaxy there are many old NSs (about 10^9), the young ($\sim 10^6$ yr) cooling NSs seems to outnumber, at the *Rosat* counts, those in phase $\mathcal{A}$. This is what emerges also from the calculation of the log N-log S distribution both of cooling and accreting stars (the last carried on with "census"; [106]). At the bright counts, the local population of cooling NSs would dominate over the log N-log S of the dimmer and warmer (less absorbed) Accretors. There is the hope that *Chandra* and *Newton* will detect them, at the flux limit of 10^{-16} erg cm^{-2} s^{-1} [106]. In support of the cooling hypothesis there is also the recent measurement[107][108] of the velocity (200 km s^{-1}) of RXJ1856 (a member of this class) indicating that this source is, most likely, not powered by accretion having a high velocity. Interestingly, the cooling hypothesis (when explored in the log N-log S plane) implies that the NS birth rate in the Solar vicinity, over the last 10^6 yr, is higher than that inferred from radiopulsar observations [106] [109]. This might be a crucial point as it may indicate that most of the young NSs may not share the observational properties of the canonical PSRs, hinting for the presence of a background of "anomalous" young cooling NSs.

About field decay in the accretion hypothesis? Decay of the field to extreme low values would imply a large number of Accretors [101], not observed, thus excluding this possibility. This indicates that, in crustal models [28] [30], electric currents need to be located deep into the crust and that the impurity parameter needs to be not exceedingly large [63], in spin-down induced models. What is still uncertain is whether the paucity is due mainly to a velocity effect than to a "fine-tuned" decay, a problem not solved yet, statistically.

While our theoretical expectation hints in favor of the cooling hypothesis there remain yet a puzzle: the long period of one of these sources, RXJ0720 [110]. If powered by accretion, RXJ0720 would have a "weak" field NS ($\mu \gtrsim 10^{26}$ G cm^3) and this would be a rather direct prove of some field decay, in isolated NSs[111][112]. Can a young cooling object have such a slow rotation? Do we have to change or view that NSs come to birth with ultra short periods? A new challenging hypothesis has been put forward [110][113] [114] that RXJ0720 is just the descendant of a highly magnetized NS (a *Magnetar*) that have suffered a severe spin down accompanied by a nonlinear decay of the field whose energy is powering the X-ray luminosity [115]. This issue remains one of the new problems of the NS physics, making the debate on the nature of these sources an even more exciting problem.

7 Conclusions

In this review we have traced the evolution of old NSs transiting through the Ejector, Propeller and Accretor phases. The NSs of our review are far from being "canonical" MSPs in light binaries, or "canonical" PSRs in the field. The ultra fastly spinning NSs, that we have recycled in binaries, are rather extreme relativistic heavy NSs: If discovered, they will unable us to probe the stellar interior in an unprecedented way, and to constrain the physics driving magnetic field decay, in interacting systems. As regard to the field NSs, the discovery of the six *Rosat* sources has just opened the possibility of unveiling "unconventional" NSs, evolving in isolation. Whether they are accreting or cooling objects is still a mystery and even more mysterious and fascinating is their possible link with "Magnetars". The study of these "unusual" NSs can open new frontiers in this already active field of research.

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References

1. Lipunov, V.M., *Astrophysics of Neutron Stars* (Springer-Verlag)
2. Shvartsman, V.F. 1971, *Soviet Astr. AJ.* 15, 337
3. Treves, A., Colpi, M. & Lipunov, V.M. 1993 *A&A*, 319
4. Lovelace, R.V.E., Romanova, M.M. & Bisnovatyi-Kogan, G. S. 1999, *ApJ*, 514, L368
5. Stella, L., White N.E. & Rosner R. 1986, *ApJ*, 308, 669
6. Campana S. *et al.* 1998 *ApJ*, 499, L65
7. Lipunov, V.M. & Popov, S.B. 1995, *AZh*, 71, 711
8. Konenkov, D. Yu. & Popov, S.B. 1997, *AZh*, 23 ,569
9. Li, X.-D. 1999, astro-ph/9903190
10. Ostriker, J.P. & Gunn, J.E. 1969, *ApJ*, 157, 1395
11. Ruderman, M. & Sutherland, P. 1975, *ApJ*, 196, 51
12. Lyne, A.G., Manchester, R.N. & Taylor, J.H. 1985, *MNRAS*,
13. Lamb, H. 1883, *Phil,Trans.Roy.Soc.London*, 174, 519
14. Canuto, V. 1969, *ApJ*, 159, 651
15. Baym, G., Pethick, C. & Pines, D. 1969, *Nature*, 224, 674
16. Chanmugam, G. & Gabriel, M. 1971, *A&A*, 16, 149
17. Narayan, R. & Ostriker, J.P. 1990, *ApJ*, 352, 222
18. Bhattacharya, D., Wijers, R.A.M.J., Hartman, J.W. & Verbunt, *A&A*, 254, 198
19. Wakatsuki, S., Hikita, A., Sato, N. & Itoh, N. 1992, *ApJ*, 392, 628
20. Lorimer, D.R. 1994, *Ph.D.Thesis*, University of Manchester, U.K.
21. Hartman, J.W., Verbunt, F., Bhattacharya, D., Wijers, R.A.M.J. 1997, *A&A*, 322, 477
22. Toscano, M. *et al.* 1999, *MNRAS*, 307, 924

23. Bhattacharya, D. & Srinivasan, G. 1986, *Curr.Sci.*, 55, 327
24. Kulkarni, S.R. 1986, *ApJ*, 306, L85
25. van den Heuvel, E.P.J., van Paradijs, J.A. & Taam, R.E. 1986, *Nature*, 322, 153
26. Verbunt, F., Wijers, R.A. & Burm, H. 1990, *A&A*, 234, 195
27. Yakovlev, D., & Urpin, V. 1980, *SvA*, 24, 303
28. Sang, Y. & Chanmugam, G. 1987, *ApJ*, 323, L61
29. Sang, Y. & Chanmugam, G. & Tsuruta, S. 1990, in *Neutron stars and their birth events*, ad. W.Kundt (Dordrecht: Kluwer), 127
30. Urpin, V.A. & Muslimov, A.G. 1992, *MNRAS*, 256, 261
31. Urpin, V.A., & Konenkov, D. 1998, *MNRAS*, 292, 167
32. Pethick, C. & Sahrling, M. 1995, *ApJ*, 453, L29
33. Ruderman, M. 1972, *ARA&A*, 10, 427
34. Srinivasan, G., Bhattacharya, D., Muslimov, A.G. & Tsygan, A.I. 1990, *Curr.Sci.*, 59, 31
35. Muslimov, A.G. & Tsygan, A.I. 1985, *Ap&SS*, 115, 43
36. Miri, M.J. & Bhattacharya, D. 1994, *MNRAS*, 269, 455
37. Camilo, F. *et al.* 2000, in *Pulsar Astronomy - 2000 and beyond*, Proc. of IAU Colloquium 177, ASP Conf.Series, in press
38. Urpin, V.A., Levshakov, S.A., & Yakovlev, D.G. 1986, *MNRAS* 219, 703
39. Wiebicke, H.J. & Geppert, U. 1996, *A&A*, 309, 203
40. Taam, R.E. & van den Heuvel, E.P.J. 1986, *ApJ*, 305, 235
41. Shibazaki, N., Murakami, T., Shaham, J. & Nomoto, K. 1989, *Nature*, 342, 656
42. Zhang, C.M., Wu, X.J. & Yang, G.C. 1994, *A&A*, 283, 889
43. Zhang, C.M. 1998, *A&A*, 330, 195
44. Wijers, R.A.M.J. 1997, *MNRAS*, 287, 607
45. Romani, R.W. 1990, *Nature*, 347, 741
46. Arons, J. & Lea, S.M. 1980, *ApJ*, 235, 1016
47. Burderi, L., King, A. & Wynn, G.A. 1996, *MNRAS*, 283, L63
48. Geppert, U., & Urpin, V. 1994, *MNRAS*, 271, 490
49. Urpin, V.A., & Geppert, U. 1996, *MNRAS*, 275, 1117
50. Urpin, V.A., & Geppert, U. 1996, *MNRAS*, 278, 471
51. Urpin, V.A., & Geppert, U. & Konenkov 1998, *A&A*, 331, 244
52. Possenti, A. 2000, *Ph.D. Thesis*, University of Bologna
53. Urpin, V.A., & Geppert, U. & Konenkov 1998, *MNRAS*,
54. Konar, S., & Bhattacharya, D., 1997, *MNRAS*, 284, 311
55. Konar, S., & Bhattacharya, D. 1999, *MNRAS*, 303, 588
56. Psaltis, D. & Lamb, F.K. 1997, Proc. Symp. *Neutron Stars and Pulsars*, Tokyo, Ed. N. Shibazaki, N. Kawai, S. Shibata, T. Kifune, Universal Academy Press, Inc.
57. White, N.E., & Zhang, W. 1997, *ApJL*, 490, L87
58. Sahrling, M. 1998, astro-ph/9804047
59. Young, E.J. & Chanmugam, G., *ApJ*, 442, L53
60. Sengupta, S. 1998, *ApJ*, 501, 792
61. Thompson, Ch. & Duncan, R. 1993, *ApJ*, 408, 194
62. Miri, M.J. 1996, *MNRAS*, 283, 1214
63. Konar, S., & Bhattacharya, D. 1999, *MNRAS*, 308, 795
64. Konenkov, D. & Geppert, U., astro-ph/9910492
65. Ruderman, M. 1991, *ApJ*, 382, 576
66. Ruderman, M. 1991, *ApJ*, 382, 587
67. Chen, K., Ruderman, M. & Zhu, T. 1998, *ApJ*, 493, 397
68. Ruderman, M., Zhu, T. & Chen, K. 1998, *ApJ*, 492, 267

69. Ruderman, M. 1995, *JA&A*, 16, 207
70. Bhattacharya, D., & van den Heuvel, E.P.J. 1991, Phys. Rep., 203, 1
71. Tauris, T.M., & Savonije, G.J, 1999, A&A, 350, 928
72. Hansen, B.M.S., & Phinney, E.S. 1997, MNRAS, 291, 569
73. Cordes, J.M., & Chernoff, D.F. 1997, ApJ, 482, 971
74. Colpi, M., Campana, S., & Treves, A. 1993, A&A, 278, 161
75. Colpi, M., Turolla, R., Zane, S., & Treves, A. 1998, ApJ, 501, 252
76. Livio, M., Xu, C., $ Frank, J. 1998, ApJ, 492, 298
77. Treves, A., Turolla, R., Zane, S., & Colpi, M. 2000, PASP, 112, 297
78. Popov, S.B., & Prokhorov, M.E. 2000, A&A, 357, 164
79. Cook, G.B., Shapiro S.L., & Teukolsky, S.A. 1994, ApJL, 423, L117
80. Burderi, L., Possenti, A., Colpi, M., Di Salvo, T., & D'Amico, N. 1999,
81. Alpar, M.A., Cheng, A.F., Ruderman, M.A., & Shaham, J. 1982, Nature, 300, 728
82. Bhattacharya, D. 1995, in X-ray Binaries, ed. W.H.G. Lewin, J. van Paradijs & E.P.J. van den Heuvel (Cambridge Univ. Press), 5
83. Possenti, A., Colpi, M., D'Amico, N., & Burderi, L. 1998, ApJL, 497, L97
84. Possenti, A., Colpi, M., Geppert, U., Burderi, L., & D'Amico, N. 1999, ApJS, 125, 463
85. van Paradijs, J. 1996, A&A, 464, L139
86. King, A.R., Frank, J., Kolb, U., & Ritter, H. 1997, ApJ, 484, 844
87. Burderi, L., & D'Amico, N. 1997, ApJ, 490, 343
88. Chen, K. & Ruderman, M. 1993, ApJ, 402, 264
89. White, N.E., & Zhang, W. 1997, ApJL, 490, L87
90. van der Klijs, M. 1998, in *The Many Faces of Neutron Stars*, eds. Alpar, Buccheri, van Paradijs, Dordrecht: Kluwer, 337
91. Andersson, N., Kokkotas, K.D., & Stergioulas, N. 1999, ApJ, 516, 307
92. Bildsten, L. 1998, ApJ, 501, L89
93. Thorsett, S.E., Chakrabarty, D. 1999, ApJ, 512, 288
94. Nice, D.J., Splaver, M.E., & Stairs, I.H. 2001, ApJ, in press (astro-ph/0010489)
95. Konar, S., & Bhattacharya, D. 1999, MNRAS, 303, 588
96. Miri, M.J., & Bhattacharya, D. 1994, MNRAS, 269, 455
97. Ruderman, M.A., Zhu, T.,& Chen, K 1998, ApJ, 492, 267
98. Sang, Y., & Chunmugam, G. 1987, ApJ, 323, L61
99. Srinivasan, G., *et al.* 1990, Curr.Sci. 59, 31
100. Urpin, V.A., Geppert U., & Konenkov, D. 1998, MNRAS, 295, 907
101. Popov, S.B., Colpi, M., Treves, A., Turolla, R., Lipunov, V.M., & Prokhorov, M.E. 2000, ApJ, 530, 896
102. Ostriker, J.P., Rees, M.J., & Silk, J. 1970, Astrophys. Lett., 6, 179
103. Treves, A.,& Colpi, M. 1991, A&A 241, 107
104. Blaes, O.,& Madau, P. 1993, ApJ, 403, 690
105. Lyne, A.G., Lorimer, D.R. 1994, Nature, 369, 127
106. Popov, S.B., Colpi, M., Prokhorov, M.E., Treves, A., & Turolla, R. 2000, to appear in ApJL
107. Walter, F., Matthews, L.D. 1997, Nature, 389, 358
108. Walter, F.M. 2000, astro-ph/0009031
109. Neühauser, R., & Trümper, J.E. 1999, A&A, 343, 151
110. Haberl, F. et al. 1997, A&A, 326, 662
111. Konenkov, D. Yu., & Popov S.B. 1997, Astronomy Letters, 23, 569
112. Wang, J. 1997, ApJ, 486, L119
113. Heyl, J.S., & Kulkarni, S.R. 1998, ApJ, 506, L61
114. Colpi, M., Geppert, U., & Page, D. 2000, ApJ, 529, L29
115. Thompson, C., & Duncan, R.C. 1996, ApJ, 473, 322