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STOCHASTIC SPIN EVOLUTION OF NEUTRON STARS

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Abstract

In this paper we present calculations of period distribution for old accreting isolated neutron stars (INSs).
At the age about few billions years low velocity INSs come to the stage of accretion. At that stage their period evolution is governed by magnetic braking and accreted angular momentum. Due to turbulence of the interstellar medium (ISM) accreted momentum can both accelerate and decelerate rotation of an INS and spin evolution has chaotic character.
Calculations show that for constant magnetic field INSs have relatively long spin periods, $\geq 10^4$–10^5 s, depending on parameters of INSs and ISM density. Due to long periods INSs have high spin up/spin down rates, which should fluctuate on a time scale about few years.

1 Introduction

Spin period is the most precisely determined parameter of a neutron star (NS). Estimates of other parameters: masses (for isolated objects), radii, temperatures, magnetic fields etc. are always model dependent. Because of that it is very important to have a clear picture of period evolution as far as this parameter is usually used to determine other characteristics of NSs. Here we try to obtain distribution of spin periods for old accreting INSs (AINSs).

AINSs are now a subject of interest in astrophysics (see Treves et al.[9)]). Probably few candidates are observed by ROSAT (Motch[7)]).

In the next section we describe the model we use to obtain period distributions and show results for the easiest case of "spin equilibrium". Then in the section 3 we present our main results for the "non-equilibrium" case and briefly discuss them in the section 4. Details of calculations can be found in Prokhorov et al.[8)].

2 "Spin equilibrium"

Previous attempts to calculate typical periods of AINSs were made by Lipunov & Popov[5)] and Konenkov & Popov[3)]. In these papers the authors do not try to obtain distributions: only characteristic periods of AINSs are derived. The authors *assume* that AINSs are in "spin equilibrium", i.e. all AINSs in these estimates have enough time to reach the stage at which magnetic braking is compensated by accretion of angular momentum. We use terms "spin equilibrium" and "non-equilibrium" in quotation-marks as far as there is no real equilibrium: period can significantly fluctuate. But the situation in general is similar to real period equilibrium in close binaries (see Ghosh & Lamb[2]), Lipunov[4])).

We start with the following equations:

$$\frac{d\omega}{dt} = F + \Phi, \; F = -\frac{k_t \mu^2}{I R_{co}^3}, \; \Phi \sim \frac{\dot{M} J}{I}. \tag{1}$$

Here I – moment of inertia of a NS, $\omega = 2\pi/p$ – spin frequency, $\mu = B\, R_*^3$ – magnetic moment of a NS, $R_{co} = (GM/\omega^2)^{1/3}$ – corotation radius, k_t – constant of order of unity, and Φ – turbulent torque, $< \Phi >= 0$. J is determined as $J =$Min $(v_t R_G, v_A R_A)$, where $v_t = 10^6 \mathrm{cm\,s^{-1}}\, (R_G/R_t)^{1/3}$ – turbulent velocity at $R = R_G$, $R_t = 2 \cdot 10^{20}$ cm, $R_G = 2GM/v^2$, $v^2 = v_s^2 + v_{sp}^2$, v_s – sound velocity, v_{sp} – spatial velocity, $R_A = \left(\mu^2/2\dot{M}\sqrt{GM}\right)^{2/7}$ – Alvfen radius, and $v_A = (GM/R_A)^{1/2}$ – Keplerian velocity at the Alfven radius. For the most reasonable parameters $J = v_t R_G$.

Now we have to introduce a kind of "diffusion coefficient", D, because due to interstellar medium (ISM) turbulence we have a kind of diffusion in the space of frequencies. This coefficient can be approximatelly determined as $D = (1/6)\left(\dot{M} J/I\right)^2 R_G/v$ (Lipunov & Popov[5])). Here $\dot{M} = \pi\, R_G^2\, \rho\, v$ – is an accretion rate.

We can determine an average spin frequency in the following way:

$$\omega_{turb}^2 = \int\limits_0^\infty \omega^4 e^{-V(|\omega|)/D} d\omega \Big/ \int\limits_0^\infty \omega^2 e^{-V(|\omega|)/D} d\omega, \; V(\mid \omega \mid) = \frac{\mu^2}{3GMI} \mid \omega \mid^3 . \tag{2}$$

Finally, $p_{turb} = 2\pi/\omega_{turb}$. For $J = v_t \cdot R_G$ we can write it as:

$$p_{turb} = 3.9 \cdot 10^8 \mu_{30}^{2/3} I_{45}^{1/3} M_{1.4}^{-26/9} n^{-2/3} v_7^{43/9} R_{t_{2,20}}^{2/9} \text{ s.} \tag{3}$$

Here $v_7 = v/(10^7\,\mathrm{cm\,s^{-1}})$, $R_{t_{2,20}} = R_t/(2 \cdot 10^{20} \mathrm{cm})$.

We note strong dependence of p_{turb} and D on v. If information on p and $\dot{p}$ is available the problem can be reversed, and one can obtain an estimate of the velocity of a AINS as it was done for stellar wind accretion in X-ray pulsars by Lipunov & Popov[6]).

Probability plotted in Fig. 1 was calculated as:

$$f(\omega, v, \mu) \propto (\mu^2/GMID)\omega^2 e^{-V/D}, \tag{4}$$

and then normalized. Here V and D are functions of v, μ, n. Note, that peaks in Fig. 1 are indeed sharp. It means, that if "spin equilibrium" can be reached, it is possible to use just one typical value, p_{turb}, for each set of parameters as it was done by Lipunov & Popov[5]) and Konenkov & Popov[3]).

3 "Non-equilibrium" calculations

In this section we calculate probability distribution for the "non-equilibrium" case. These distributions are calculated for magnetic moments with Gaussian distribution in logarithmic scale with central value $\lg(\mu_0) = 30.06$ and $\sigma = 0.32$ (see Colpi et al. 2001[1]) for details on magnetic evoluyion of NSs). Velocities of AINS are taken from the Maxwellian distribution with a mean velocity 200 km s^{-1}.

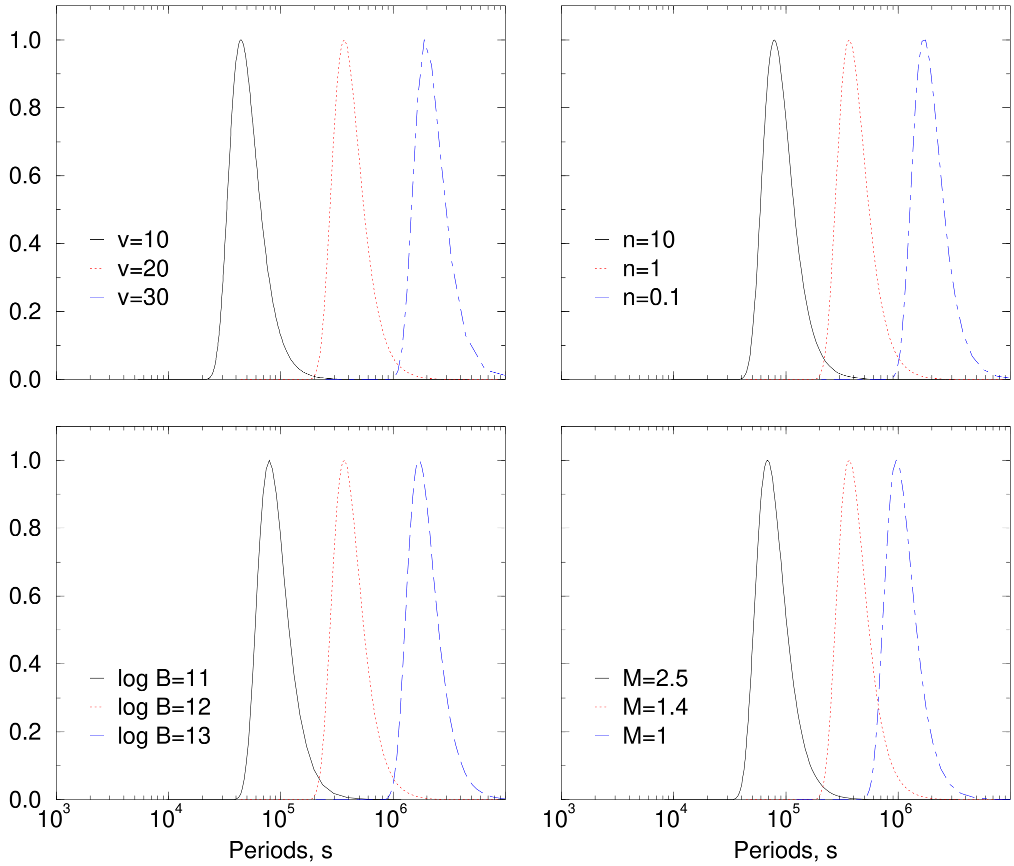

Figure 1: Period distributions in equilibrium for different parameters. In each of the four plots we vary one parameter: n, v, M, B. Results are normalized to unity at the maximum.

We solved numerically the following differential equation:

$$df/dt = A/\omega^2 \partial(\omega^4 f)/\partial\omega + D/\omega^2 \partial/\partial\omega \left(\omega^2\, \partial f/\partial\omega\right), \; A = \mu^2/GMI. \tag{5}$$

After initial parameters of an INS are chosen from the distributions described above we check if this NS can reach the accretion stage in 10^{10} yrs. To do it we calculate time which it spends as Ejector: $t_E \approx 10^9 n^{-1/2} v_6 \mu_{30}^{-1}$yrs, $v_6 = v/10^6$ cm s^{-1}. We neglect the stage of Propeller, as far as for constant field it is much shorter than the stage of Ejector (Lipunov & Popov[5])).

In Fig. 2 we present a curve for $n = 1$ cm^{-3} and selected INS parameters: $\mu = 2 \cdot 10^{29}$ G cm^3, $v_{sp} = 10$ km s^{-1}. In Fig. 3 we show our final results for Maxwellian velocity distribution and log-Gaussian magnetic field distribution for two values of the ISM density.

4 Discussion

After an INS come to the stage of accretion it is controlled by two processes (see eq.1): magnetic spin-down and turbulent spin-up/spin-down. Initially magnetic spin-down is more significant, but at some period, p_{cr}, these two processes become comparable. For longer periods an INS will be governed mainly by turbulent forces. One can obtain the following formula for p_{cr}: $p_{cr}^2 = (4\pi^2\mu^2)/(GM\dot{M}J)$. An INS reach p_{cr} in $\Delta t \sim 10^5$–10^7 yrs after onset of accretion: $\Delta t = (I\sqrt{GM})/(\mu\sqrt{\dot{M}J})$.

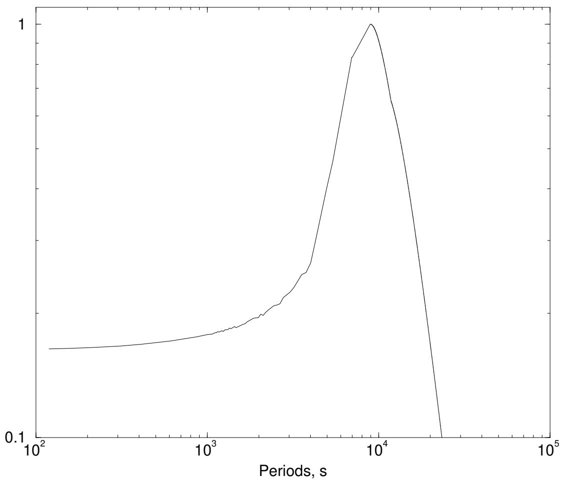

Figure 2: Period distribution for $\mu = 2 \cdot 10^{29}$ G cm^3, $v_{sp} = 10$ km s^{-1}, $n = 1$ cm^{-3}. Results are normalized to unity at the maximum.

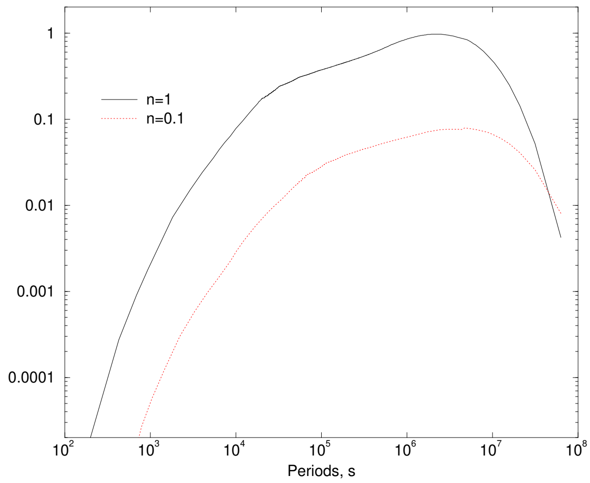

Figure 3: Period distributions for $n = 1$ cm^{-3} and $n = 0.1$ cm^{-3}. Results are normalized to unity at the maximum of the highest curve.

For field decay picture should be completely different (see for example Konenkov & Popov[3)]). AINSs with decayed field can appear as pulsating sources with periods about 10 s and $\dot{p}$ about 10^{-13} s/s. As an INS passes through turbulent cells a value and a sign of $\dot{p}$ will fluctuate on a time scale $R_G/v_{sp} \simeq 11.8\, v_{sp}/(10\,\mathrm{km\,s^{-1}})$ yr.

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